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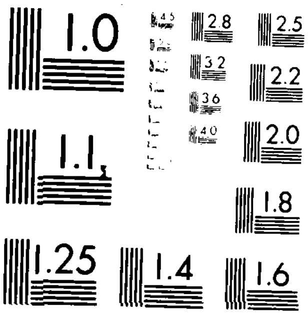

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A LATERAL-DIRECTIONAL CONTROLLER FOR HIGH-ANGLE-OF-ATTACK FLIGHT

by

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ABSTRACT

A digital flight control system based on microprocessor technology has been designed, developed, and flight tested using the Avionics Research Aircraft (ARA) at Princeton University. The control system utilizes the existing microprocessor system available in the aircraft's fly-by-wire control system. The command and stability augmentation control law was developed using modern control theory and is incorporated into existing flight control computer programs. Development of the model and control law, the gain scheduling procedure, and the flight test results are presented.

The objective of the study was to provide lateral-directional stability during high-angle-of-attack flight and into the stall regime. Flight test results show that it is indeed possible to design a control system which will eliminate lateral-directional instabilities and do so at a level higher than the pilot was able to attain. In addition, gain scheduling proved to be capable of providing satisfactory control throughout a wide range of flight conditions. Additional work, however, is required to correct a number of control system inadequacies before the control system can become operational.

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This thesis carries 1583-T in the records of the Department of Mechanical and Aerospace Engineering.

TABLE OF CONTENTS

TABLE OF CONTENTS - Continued

LIST OF TABLES

LIST OF FIGURES

Chapter I

INTRODUCTION

1.1 DESCRIPTION OF THE PROBLEM

The study of aircraft in high angles of attack is of critical importance to the understanding of aircraft flight and to design for aircraft safety. It is in this regime that the maximum lift is reached and stall encountered -- one of the primary reasons for aircraft accidents. By understanding the high angle-of-attack regime, it might be possible to design safety features into an aircraft which could delay the onset of stall, warn the pilot of an impending stall, or enable the pilot to recover more quickly. This topic has been the subject of ongoing study at the Princeton Flight Research Laboratory (FRL).

While stall is primarily a problem of the longitudinal mode of the aircraft, the lateral-directional mode can be seriously affected in the regime near stall. With lateral-directional controls greatly reduced, the aircraft can experience instabilities such as wing rock. Indeed, these instabilities have been noted in the research aircraft at Princeton (Ref. 1). In addition, it has been found that increasing the throttle tends to aggravate the unstable condition. The lateral-directional instabilities can cause difficulties in obtaining data in the high angle-of-attack regime and thus severely hamper efforts in dealing with aircraft stall. Therefore, a lateral-directional command augmentation system (CAS) would be helpful to steady the aircraft and to isolate the critical aspects of high angle-of-attack flight from lateral-directional disturbances. The topic of this study, then, is the design and testing of a lateral-directional CAS capable of operating at all flight conditions which the aircraft might encounter but with special emphasis on high angle-of-attack conditions.

The FRL has at its disposal two research aircraft, [illegible] of which is the Avionics Research Aircraft (ARA) Navion. The Navion has been tested extensively in the wind tunnel at NASA's research facility at Hampton, Virginia and the results are summarized in NASA TN D-5857 (Ref. 2). The availability of the data simplified the process of developing a model, and the presence of the aircraft enhanced the ability to validate the control system once it was designed.

The ARA has been fitted with a microprocessor for digital flight control. The CAS, then, was designed to be digital to take advantage of the microprocessor. In addition, FRL has at its disposal a ground-based microprocessor system for software development and testing.

The CAS itself was designed as a two-input, two-output, single command mode controller. Lateral stick and pedals were selected as inputs and were scaled to be roll rate and sideslip commands, respectively. The outputs were commands to the ailerons and rudder. Three longitudinal variables -- angle of attack, throttle setting, and dynamic pressure -- were selected as the flight condition. Since the CAS was required to work at all flight conditions, the gains were scheduled with these three variables.

1.2 ORGANIZATION OF THE THESIS

The thesis is presented to cover the critical steps that were taken to design and test the CAS. In particular, Chapter 2 deals with the development of the model of the AKA. In addition, the linearization procedure is discussed. Finally, a set of open-loop results is presented for comparison with known aircraft behavior.

Chapter 3 discusses the calculation of the control gains using linear-quadratic, sampled-data regulator theory. Singular command equilibrium is discussed, as is the calculation of the optimal gains. In addition, the control law is developed and presented. Finally, results are included showing the selection of the continuous-time weighting matrices, a detailed description of the CAS performance for a nominal flight condition, and a summary of closed-loop simulation results for a variety of flight conditions.

Chapter 4 discusses the gain scheduling process. The form of the gain equations is discussed, as is the solution of the flight condition functions which make up the gain equations. Also included is the computation of the gain coefficient matrices. Finally, results of a simulation using the gain schedules are presented.

Chapter 5 covers the flight tests. The microprocessor system used and the development of software for CAS implementation are discussed. In addition, ground tests performed before flight tests are covered. Finally, the flight test procedures and a detailed analysis of the results are included.

Chapter 6 presents some conclusions and possible recommendations for further work in the area of gain scheduling.

Tha appendices cover the extraneous areas of study which were required for successful completion of the project. Appendix A presents the aircraft data used in the development and testing of the model. Appendix B summarizes the gain calculation software while Appendix C presents the microprocessor software. References lists the sources of information used during the course of this research.

Chapter II

DEVELOPMENT OF THE MODEL

The requirements of the control system specified that the CAS should provide satisfactory control for not just one flight condition, but for the whole spectrum of flight conditions the aircraft might encounter. This in turn required the development of a model which could accurately predict aircraft behavior throughout this same range of flight conditions. In other words, the model was developed not only as a function of the state and controls, but also of the flight condition.

The model could have taken two forms. The first stores the aerodynamic data in a table. To use this method, the model would be looked up for the particular flight condition from a set of tables containing a number of possible models. Since a model for every possible flight condition combination could not be listed, some flight conditions would have to be approximated by the closest model available or by interpolation between models. This idea was rejected as requiring too much computer storage and excessive run time.

The second choice was to find a set of polynomials to calculate the model based on the specified value of the flight condition. There is a considerable amount of aerodynamic data from which the polynomials could be derived. All that was required was to reduce the available data to a set of equations. This was the method selected in this study.

The experimental data were compiled in NASA TN D-5857 (Ref. 2) and that which were used are summarized in Appendix A. The NASA report included data on stability and control derivatives. Experimental values of rotary derivatives were given in NASA TN D-6643 (Ref. 3) but these were considered insufficient since they did not include variations of these variables with the changing flight condition. Instead, the USAF STABILITY AND CONTROL DATCOM (Ref. 4) methods were used to derive the set of rotary derivative equations, and the values available in TN D-6643 were used for comparison.

The selection of the flight condition variables was made on the basis of available data and control system needs. Since the control system was to be angle-of-attack sensitive, angle of attack, a, was chosen as a variable. Fortunately, the flight data presented the variations of the lateral-directional parameters against variations in angle of attack. Since the application of throttle resulted in aggravating the unstable condition, throttle setting, T_c, as chosen as a flight condition variable. Data were also available showing the effect of throttle setting on the lateral-directional parameters. Finally, given the overall effect of dynamic pressure, $\bar{q}$ (velocity and altitude effects), it was chosen as the final flight condition variable. The ranges of each variable were based on typical operating conditions and available data: angle of attack, -4 to 24 degrees; throttle setting (representing nondimensional value of thrust, $T/\bar{q}S$), .03 to .23; and dynamic pressure, which was based on typical velocities for the aircraft of 100 to 200 feet per second and an altitude of 5000 feet.

This chapter covers the model development process from the linearization of the nonlinear equations to the data reduction and formulation of the nondimensional force and moment coefficients. Open-loop results for 27 different flight conditions are presented to help verify the model.

2.1 NONLINEAR, TIME-VARYING EQUATIONS

The nonlinear, lateral-directional equations can easily be found in the literature (Ref. 3) and are summarized as follows:

$$\dot{y} = u\cos(\theta_o)\sin(\Psi)+v(\sin(\phi)\sin(\theta_o)\sin(\Psi)-\cos(\phi)\cos(\Psi))$$

$$w(\cos(\phi)\sin(\theta_o)\sin(\Psi)-\sin(\phi)\cos(\Psi)) \qquad (2\text{-}1)$$

$$\dot{v} = -ru + pw + g\cos(\theta_o)\sin(\phi) + (\bar{q}S/m)C_y \qquad (2\text{-}2)$$

$$\dot{p} = (\bar{q}Sb/I_x)C_l \qquad (2\text{-}3)$$

$$\dot{r} = (\bar{q}Sb/I_z)C_n \qquad (2\text{-}4)$$

$$\dot{\phi} = p + (q_o\sin(\phi) + r\cos(\phi))\tan(\theta_o) \qquad (2\text{-}5)$$

$$\dot{\Psi} = q_o\sin(\phi)\sec(\theta_o) + r\cos(\phi)\sec(\theta_o) \qquad (2\text{-}6)$$

The Ψ angular position and the y position variables give information on the aircraft with respect to some fixed axis system, but they have no effect on the aircraft's stability. Hence, these two variables were dropped, leaving a fourth-order model. In addition, the lateral velocity variable, v, does not have as much physical meaning to the pilot as sideslip angle, B. To get the sideslip rate equation, the $\dot{v}$ equation was divided by the total velocity, V_o. Finally, the equations were rotated through the angle-of-attack (to the stability axis system). Hence, the final nonlinear equations of motion were:

$$\dot{r} = (\bar{q}Sb/I_z)C_n \tag{2-7}$$

$$\dot{B} = (1/V_o)(-ru + g\cos(\gamma_o)\sin(\phi) + (\bar{q}S/m)C_y \tag{2-8}$$

$$\dot{p} = (\bar{q}S/I_x)C_l \tag{2-9}$$

$$\dot{\phi} = p + (q_o\sin(\phi) + r\cos(\phi))\tan(\ _o) \tag{2-10}$$

These equations were rewritten as a vector differential equation:

$$\underline{x} = \underline{f}(\underline{x}(t),\ \underline{u}(t)) \tag{2-11}$$

where $\underline{x}$ is the state vector (r, B, p, ϕ) and $\underline{u}$ is the control vector (dR, dA).

2.2 LINEARIZATION

The development of a control system based on the nonlinear equations of motion would be quite difficult. Therefore, it was imperative that the equations be linearized. This was done using a Taylor series approximation. The approximation was taken about a trim point -- the forces and moments add up to zero, and the aircraft is at equilibrium. The Taylor series approximation of the nonlinear equations, $\underline{f}$, is as follows:

$$\dot{\underline{x}} = \underline{f}(\underline{x}_o,\underline{u}_o) + \frac{\partial \underline{f}(\underline{x}_o,\underline{u}_o)}{\partial \underline{x}} \Delta\underline{x} + \frac{\partial \underline{f}(\underline{x}_o,\underline{u}_o)}{\partial \underline{u}} \Delta\underline{u} + \frac{\partial^2 \underline{f}(\underline{x}_o,\underline{u}_o)}{\partial \underline{x}^2} \Delta\underline{x}^2 + \frac{\partial^2 \underline{f}(\underline{x}_o,\underline{u}_o)}{\partial \underline{u}^2} \Delta\underline{u}^2 + \ldots \quad (2\text{-}12)$$

where $\underline{x}_o$ and $\underline{u}_o$ are the trim values of the states and controls, respectively. By dropping the higher order terms and rearranging, the approximation becomes:

$$\underline{x}_o = \underline{f}(\underline{x}_o,\underline{u}_o) \quad (2\text{-}13)$$

$$\Delta\dot{\underline{x}} = \frac{\partial \underline{f}(\underline{x}_o,\underline{u}_o)}{\partial \underline{x}} \Delta\underline{x} + \frac{\partial \underline{f}(\underline{x}_o,\underline{u}_o)}{\partial \underline{u}} \Delta\underline{u} \quad (2\text{-}14)$$

The $\underline{x}_o$ equation represents the nominal trim solution and the $\underline{x}$ equation represents the deviations from trim. It was this perturbation equation that was of the most interest. The derivative terms form a Jacobian matrices and were redefined as:

$$F = \frac{\partial \underline{f}(\underline{x}_o,\underline{u}_o)}{\partial \underline{x}} \qquad G = \frac{\partial \underline{f}(\underline{x}_o,\underline{u}_o)}{\partial \underline{u}}$$

The final linearized perturbation equations of motion (in matrix form) were as follows:

$$\Delta\dot{\underline{x}} = F(t)\,\Delta\underline{x} + G(t)\,\Delta\underline{u} \quad (2\text{-}15)$$

where F is the system dynamics matrix and G is the control dynamics matrix.

The nondimensional force and moment coefficients (C_y, C_n, C_l) are functions of the states and controls and were rewritten in linearized form as:

$$C_y = C_{y_o} + C_{y_B}\Delta B + (^b/(2V_o))C_{y_r}\Delta r + (^b/(2V_o))C_{y_p}\Delta p + C_{y_{dR}}\Delta dR + C_{y_{dA}}\Delta dA \quad (2\text{-}16)$$

$$C_n = C_{n_o} + C_{n_B}\Delta B + (^b/(2V_o))C_{n_r}\Delta r + (^b/(2V_o))C_{n_p}\Delta p + C_{n_{dR}}\Delta dR + C_{n_{dA}}\Delta dA \qquad (2\text{-}17)$$

$$C_l = C_{l_o} + C_{l_B}\Delta B + (^b/(2V_o))C_{l_r}\Delta r + (^b/(2V_o))C_{l_p}\Delta p + C_{l_{dR}}\Delta dR + C_{l_{dA}}\Delta dA \qquad (2\text{-}18)$$

where the equation coefficients are stability derivatives (C_{Y_B}, C_{n_B}, C_{l_B}), rotary derivatives (C_{Y_p}, C_{n_p}, C_{l_p}, C_{Y_r}, C_{n_r}, C_{l_r}), and control derivatives ($C_{Y_{dR}}$, $C_{n_{dR}}$, $C_{l_{dR}}$, $C_{Y_{dA}}$, $C_{n_{dA}}$, $C_{l_{dA}}$). The linearized equations in matrix form became (in dimensional terms):

$$F = \begin{bmatrix} N_r & N_B & N_p & 0 \\ Y_r/V_o - 1 & Y_B/V_o & Y_p/V_o & g*\cos(\gamma_o)/V_o \\ L_r & L_B & L_p & 0 \\ \tan(\gamma_o) & 0 & 1 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} N_{dR} & N_{dA} \\ Y_{dR}/V_o & Y_{dA}/V_o \\ L_{dR} & L_{dA} \\ 0 & 0 \end{bmatrix}$$

where $(\ )_B = C_{(\ )_B}{}^{\bar{q}S}/m$, etc.

In addition to the equations of motion, an output equation was required to keep track of certain quantities over which explicit control was desirable. This was done using an output equation of the form:

$$\underline{y}(t) = \underline{h}(\underline{x}(t), \underline{u}(t)) \qquad (2\text{-}19)$$

Performing the Taylor series approximation and redefining, the perturbation output equation became:

$$\Delta \underline{y} = H_x \Delta \underline{x} + H_u \Delta \underline{u} \qquad (2\text{-}20)$$

where

$$H_x = \frac{\partial \underline{h}(\underline{x}_o,\underline{u}_o)}{\partial \underline{x}} \qquad H_u = \frac{\partial \underline{h}(\underline{x}_o,\underline{u}_o)}{\partial \underline{u}}$$

The selection of $\underline{h}$ is discussed later.

2.2.1 Calculation of the Trim Condition

At the trim condition, all of the forces and moments sum to be zero; hence, the state is in equilibrium ($\Delta \underline{x} = \underline{0}$). Therefore, to solve for the trim condition, $\underline{f}$ was set to zero and the states computed.

$$0 = (\bar{q}Sb/I_z)C_n \qquad (2\text{-}21)$$

$$0 = (1/V_c)(-ru_o + g\cos(\gamma_c)\sin(\phi) + (\bar{q}S/m)C_y)) \qquad (2\text{-}22)$$

$$0 = (\bar{q}Sb/I_x)C_l \qquad (2\text{-}23)$$

$$0 = p + (q_c\sin(\phi) + r\cos(\phi))\tan(\gamma_c) \qquad (2\text{-}24)$$

Multiplying by terms that were unchanging, equations (2-21) to (2-24) simplified to:

$$0 = C_n \qquad (2\text{-}25)$$

$$0 = -ru_o + g\cos(\gamma_c)\sin(\phi) + (\bar{q}S/m)C_y \qquad (2\text{-}26)$$

$$0 = C_l \qquad (2\text{-}27)$$

$$0 = p + (q_c\sin(\phi) + r\cos(\phi))\tan(\gamma_c) \qquad (2\text{-}28)$$

With no initial roll rate or yaw rate and zero C_n and C_1, the nominal values of p and r were zero. Thus, from (2-16), (2-17), and (2-18), equations (2-25) to (2-28) simplified to:

$$0 = C_{n_o} + C_{n_B}\Delta B + C_{n_{dR}}\Delta dR + C_{n_{dA}}\Delta dA \qquad (2\text{-}29)$$

$$0 = C_{y_o} + C_{y_B}\Delta B + C_{y_{dR}}\Delta dR + C_{y_{dA}}\Delta dA+((gm)/(\bar{q}S))\cos(\gamma_o)\sin(\phi) \qquad (2\text{-}30)$$

$$0 = C_{1_o} + C_{1_B}\Delta B + C_{1_{dR}}\Delta dR + C_{1_{dA}}\Delta dA \qquad (2\text{-}31)$$

$$0 = 0 \qquad (2\text{-}32)$$

By examining (2-29) to (2-31), it was apparent that the aircraft could could have a trim sideslip and zero roll angle, a trim roll angle and zero sideslip, or a combination of roll angle and sideslip. In every case the trim controls were non-trival.

For this study, roll angle was set to zero, leaving a trim sideslip. Using this and manipulating equations (2-29) to (2-31), the following matrix solution for B, dR, and dA was obtained:

$$\begin{bmatrix} \Delta B \\ \Delta dR \\ \Delta dA \end{bmatrix} = \begin{bmatrix} C_{n_B} & C_{n_{dR}} & C_{n_{dA}} \\ C_{y_B} & C_{y_{dR}} & C_{y_{dA}} \\ C_{1_B} & C_{1_{dR}} & C_{1_{dA}} \end{bmatrix}^{-1} \begin{bmatrix} -C_{n_o} \\ -C_{y_o} \\ -C_{1_o} \end{bmatrix} \qquad (2\text{-}33)$$

The solution of (2-33) provided the trim condition about which the equations were linearized and the perturbations measured.

2.2.2 Selection of the Command Vector

While the selection of the command vector, $\underline{h}$, has no effect on the eventual stability of the closed loop system, it can drastically affect the response of the system. If the command vector is chosen such that the state contains an integral of one of the command vector elements, then the response will be singular. Singular command equilibrium does not necessarily imply state and control equilibrium. Furthermore, computation of the control gains will result in a set of integral feedforward elements, known as proportional-integral filtering, in addition to the usual feedforward and feedback gains.

Comparison of singular versus nonsingular command modes shows that the singular command mode has the effect of adding a third state to the two-state command vector, namely the integral state (Ref. 5). Selection of the singular command mode, then, appears to be a wise choice and the extra work involved in computation of such a control law is made up for in the improved control over the response of the system.

In this study, the command vector was chosen as roll rate (p) and sideslip angle (β)--both variables considered valuable in aircraft control. In addition, since the state vector contains roll angle (φ), an integral of roll rate, control of that same variable was effectively added. The control law configuration, then, is represented in Figure 1 .

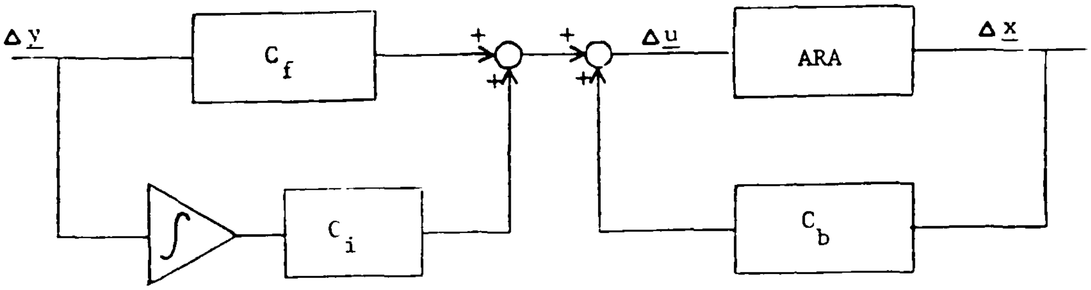


Figure 1: Control Law Configuration

2.3 NONDIMENSIONAL FORCE AND MOMENT COEFFICIENTS

Perhaps the most important part of modelling is the determination of the nondimensional coefficients--side force (C_y), yaw moment (C_n), and roll moment (C_l). The coefficients are affected by the states, the controls, and the flight condition, and they are nonlinear. The task, then, was to reduce the available data to a set of polynomials from which the coefficients could be calculated.

In the form of equations (2-16) to (2-18), the nondimensional coefficients were linear functions of the state and controls. However, the coefficients of these equations (which for sake of clarity, shall be called coefficient components) were nonlinear functions of the flight condition. Equations for these components were derived which would allow the coefficients to be calculated for any specified flight condition and state and control vectors. The coefficient components, for the purposes of this report, are broken down and discussed in four catagories:

the constant components (C_{y_o}, C_{n_o}, C_{l_o}); the stability derivative components (C_{y_B}, C_{n_B}, C_{l_B}); the rotary derivative components (C_{y_p}, C_{n_p}, C_{l_p}, C_{y_r}, C_{n_r}, C_{l_r}); and the control derivative components ($C_{y_{dR}}$, $C_{n_{dR}}$, $C_{l_{dR}}$, $C_{y_{dA}}$, $C_{n_{dA}}$, $C_{l_{dA}}$).

The aircraft for which this control system was designed was tested extensively in the wind tunnel and the data presented in NASA TN D-5857. The NASA report lists data from which the constant components, the stability derivative components, and the control derivative components could be derived directly. The method used to reduce the data was generally the same in each case and is discussed in detail in the constant component section. Since there was a considerable amount of data on these components, certain assumptions were made to to facilitate the data reduction. In particular, throttle effects were considered linear, velocity was assumed to affect only the dimensional components (through dynamic pressure), and state and control variables were assumed to affect the coefficients linearly. The validity of each of these assumptions is discussed below.

The data which were available for the other components were not available for the rotary derivatives. The report NASA TN D-6643 did present some experimental values of these components but for only one flight condition. For this study, it did not seem wise to assume that the rotary derivatives would stay constant throughout the range of flight conditions. Therefore, the USAF DATCOM methods were used, making it possible to reduce even the rotary derivative components to functions of the flight condition.

2.3.1 Constant Components

The static flight data included in NASA TN D-5857 were presented as sets of curves showing how each coefficient changes as the state, controls, and flight condition vary. In general, there was one plot per throttle setting with each plot having several curves plotted against angle of attack. The curves each correspond to a different value of a state or control variable. Each curve was plotted with ten explicit data points (corresponding to angles of attack of -4, 0, 4, 8, 12, 14, 16, 18, 20, and 22 deg). The first step taken for data reduction was to make a table of eight data points (the explicit points at -4, 0, 4, 8, 12, 16, and 20 deg angle of attack and one at 24 deg found by extrapolating the curves) at each throttle position and angle of attack. The data are presented in Appendix A.

The data were reduced to a linear equation where the coefficients of that equation were functions of angle of attack only. The form of these equations was:

$$C_{y_o} = C_{y_{o_T}} T_c + C_{y_{o_c}} \qquad (2-34)$$

$$C_{n_o} = C_{n_{o_T}} T_c + C_{n_{c_o}} \qquad (2-35)$$

$$C_{l_o} = C_{l_{o_T}} T_c + C_{l_{c_o}} \qquad (2-36)$$

The value of the slope and constant terms were found by doing a linear regression at each angle of attack. The assumption of throttle linearity proved to be valid in this case. Curves were fitted to the slope and constant points, which were functions of angle of attack alone. (In the case of C_{l_o}, however, it was apparent that curve fitting would be very difficult for the slope term, $C_{l_{o_\eta}}$. Therefore, an average C_{l_o} was calculated for the three throttle settings used, essentially neglecting throttle effects).

The curve fitting method used assumed a solution in the form of an n-th order polynomial:

$$C = A_n a^n + A_{n-1} a^{n-1} + \ldots + A_1 a^1 + A_0 \qquad (2\text{-}37)$$

In this case, a 7-th order equation was used such that eight values of the dependent and independent variables were required. A matrix equation was then set up in the following manner:

$$\begin{bmatrix} C(a_1) \\ C(a_2) \\ \cdot \\ \cdot \\ \cdot \\ C(a_8) \end{bmatrix} = \begin{bmatrix} 1 & a_1 & a_1^2 & \ldots & a_1^7 \\ 1 & a_2 & a_2^2 & \ldots & a_2^7 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 1 & a_8 & a_8^2 & \ldots & a_8^7 \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ \cdot \\ \cdot \\ \cdot \\ A_7 \end{bmatrix} \qquad (2\text{-}38)$$

where $C(a_i)$ is the value of the coefficient at angle of attack, a_i. More simply:

$$\underline{C} = M \ \underline{A} \qquad (2\text{-}39)$$

To solve for the coefficients, all that was required was a matrix inversion and multiplication by the coefficient vector:

$$\underline{A} = M^{-1} \; \underline{C} \qquad (2\text{-}40)$$

(Since the same eight angles of attack -- -4, 0, 4, 8, 12, 16, 20, 24 deg -- always were used whether calculating the constant components, the stability derivative components, or the control derivative components, M and hence M^{-1} were always the same. Thus, it was necessary only to assemble $\underline{C}$ to find the coefficients, $\underline{A}$, for any component coefficient).

The final reduced constant component equations are summarized as follows:

$$C_{y_{c_1}} = 1.094E\text{-}8\ a^7 - 7.104E\text{-}7\ a^6 + 1.630E\text{-}5\ a^5 - 1.496E\text{-}4\ a^4 + 2.979E\text{-}4\ a^3 + 2.448E\text{-}3\ a^2 - 1.715E\text{-}2\ a + 0.0$$

$$C_{y_{c_0}} = -1.996E\text{-}9\ a^7 + 1.298E\text{-}7\ a^6 - 2.974E\text{-}6\ a^5 + 2.713E\text{-}5\ a^4 - 5.525E\text{-}5\ a^3 - 4.614E\text{-}4\ a^2 + 1.812E\text{-}3\ a + 0.0$$

$$C_{n_{c_1}} = 5.553E\text{-}9\ a^7 - 3.259E\text{-}7\ a^6 + 6.695E\text{-}6\ a^5 - 5.416E\text{-}5\ a^4 + 7.633E\text{-}5\ a^3 + 1.025E\text{-}3\ a^2 - 4.504E\text{-}3\ a - 0.03281$$

$$C_{n_{c_0}} = 3.817E\text{-}10\ a^7 - 2.568E\text{-}8\ a^6 + 6.104E\text{-}7\ a^5 - 5.881E\text{-}6\ a^4 + 1.520E\text{-}5\ a^3 + 8.592E\text{-}5\ a^2 - 5.593E\text{-}4\ a + 0.001822$$

$$C_{l_0} = -7.099E\text{-}10\ a^7 + 4.458E\text{-}8\ a^6 - 9.659E\text{-}7\ a^5 + 7.877E\text{-}6\ a^4 - 6.971E\text{-}6\ a^3 - 1.637E\text{-}4\ a^2 + 2.167E\text{-}4\ a + 0.00717$$

2.3.2 Stability Derivative Components

The method for finding the stability derivative component equations was essentially the same as for the constant components except that one initial step -- the computation of the stability derivatives -- was included. The data were presented as one plot per throttle setting with seven curves per plot. The curves (each corresponding to a different value of sideslip -- -15, -10, -5, 0, 5, 10, and 15 deg) showed how the coefficients varied with angle of attack. Once again, eight points per curve were selected, and these are presented in Appendix A.

The first task was to compute the stability derivatives for each angle of attack and throttle setting. Since the coefficients were assumed linear with respect to the states, a linear regression was used for this task. The assumption of linear sideslip effects was valid except for C_n and C_l at high angles of attack. The stability derivatives were reduced to the form:

$$C_{y_B} = C_{y_{B_T}} T_c + C_{y_{B_o}} \qquad (2\text{-}47)$$

$$C_{n_B} = C_{n_{B_T}} T_c + C_{y_{B_o}} \qquad (2\text{-}48)$$

$$C_{l_B} = C_{l_{B_T}} T_c + C_{y_{B_o}} \qquad (2\text{-}49)$$

Since only two throttle settings were given in the data, no analysis of throttle linearity was required. Once again, linear regression was used to determine the equations for the stability derivatives. The curve fitting method previously discussed was used to determine the equations for the slope and constant terms. The final reduced stability derivative component equations are summarized as follows:

$$C_{y_{E_1}} = 1.274E\text{-}9\ a^7 - 8.630E\text{-}8\ a^6 + 2.089E\text{-}6\ a^5 - 2.044E\text{-}5\ a^4 + 4.791E\text{-}5\ a^3 + 3.116E\text{-}4\ a^2 - 1.607E\text{-}3\ a - 0.0188$$

$$C_{y_{E_o}} = 2.380E\text{-}4\ a - 0.01249 \quad (C_{y_{B_o}} \text{ was assumed a straight line})$$

$$C_{n_{E_1}} = -2.004E\text{-}11\ a^7 + 2.584E\text{-}9\ a^6 - 8.590E\text{-}8\ a^5 + 1.123E\text{-}6\ a^4 - 4.376E\text{-}6\ a^3 - 2.051E\text{-}5\ a^2 + 3.196E\text{-}4\ a + 0.001$$

$$C_{n_{E_o}} = 3.033E\text{-}12\ a^7 - 4.747E\text{-}10\ a^6 + 1.815E\text{-}8\ a^5 - 2.599E\text{-}7\ a^4 + 1.068E\text{-}6\ a^3 + 3.907E\text{-}6\ a^2 - 6.049E\text{-}5\ a + 0.002$$

$$C_{l_{E_1}} = -3.283E\text{-}10\ a^7 + 2.310E\text{-}8\ a^6 - 5.742E\text{-}7.\ a^5 + 5.038E\text{-}6\ a^4 - 1.109E\text{-}5\ a^3 - 9.987E\text{-}5\ a^2 + 4.282E\text{-}4\ a - 0.00100$$

$$C_{l_{E_o}} = 6.697E\text{-}11\ a^7 - 4.406E\text{-}9\ a^6 + 1.063E\text{-}7\ a^5 - 1.012E\text{-}6\ a^4 + 1.851E\text{-}6\ a^3 + 1.483E\text{-}5\ a^2 - 1.463E\text{-}5\ a - 0.00155$$

2.3.3 Rotary Derivative Components

Since no flight test or wind tunnel data were available showing the variation of the nondimensional coefficients with changes in roll rate and yaw rate, the <u>USAF STABILITY AND CONTROL DATCOM</u> methods were used to find the rotary derivatives. In general, the rotary derivatives are functions of the asymmetrical distribution of lift and drag over the wing panels caused by rolling and yawing. Because of this, the nondimensional lift and/or drag coefficients, C_L and C_D, were found in every equation for the rotary derivative components. Therefore, these coefficients were derived first, as functions of the flight condition. Each derivative is discussed in turn.

Every rotary derivative experiences compressibility effects at sufficiently high Mach numbers. However, since the velocity of the aircraft is decidedly subsonic ($M < .2$), a constant Mach number was used in the equation formulation. Since the static data in the NASA TN D-5857 was taken at 95 feet/second, the equivalent Mach number of .085 was used.

A number of other terms were common to all the rotary derivative component equations, as well. One of these terms, $(\Delta C_{y_\beta})_{v(WBH)}$, is a tail-body sideslip derivative specified by:

$$(\Delta C_{y_\beta})_{v(WBH)} = -k(C_{L_\alpha})_{v(BH)}(1+\partial\sigma/\partial\beta)(q_v/q_w)(S_v/S_w) \qquad (2-56)$$

Using aircraft constants and charts available in USAF DATCOM, $(\Delta C_{y_\beta})_{v(WBH)}$ reduced to:

$$(\Delta C_{y_\beta})_{v(WBH)} = -0.0181$$

In addition, a number of other constants relating to the aircraft (e.g. z, z_p, l_p, b_w, b_H, S_w, S_H) were present in the equations and can be found in Appendix A.

Longitudinal Nondimensional Coefficients

Since neither C_L nor C_D are significantly affected by the lateral-directional parameters, only variations with respect to the flight condition were considered. Variations with respect to changes in the longitudinal states and controls were not required. The data values are listed in Appendix A.

First, the coefficients were reduced to linear functions of the thrust coefficient:

$$C_L = C_{L_T}\ T_c + C_{L_o} \qquad (2\text{-}57)$$

$$C_D = C_{D_T}\ T_c + C_{D_o} \qquad (2\text{-}58)$$

where the coefficients of these equations were found at each angle of attack using linear regression. The assumption of linear throttle effects was valid in this case.

The curve fitting method previously discussed was used to find the equations of the slope and constant terms for the longitudinal coefficients. The equations are summarized as follows:

$$C_{L_T} = 2.780E\text{-}8\ a^7 - 1.810E\text{-}6\ a^6 + 4.299E\text{-}5\ a^5 - 4.463E\text{-}4\ a^4 + 1.530E\text{-}3\ a^3 + 6.761E\text{-}3\ a^2 - 1.321E\text{-}3\ a + 0.357$$

$$C_{L_o} = -5.571E\text{-}10\ a^7 + 8.952E\text{-}8\ a^6 - 3.684E\text{-}6\ a^5 + 5.420E\text{-}5\ a^4 - 2.531E\text{-}4\ a^3 - 9.214E\text{-}4\ a^2 + 0.06337\ a + 0.121$$

$$C_{D_T} = -1.401E\text{-}8\ a^7 + 1.045E\text{-}6\ a^6 - 2.756E\text{-}5\ a^5 + 2.968E\text{-}4\ a^4 -$$

$$9.036E-4\ a^3 - 0.003391\ a^2 + 0.02157\ a - 1.053$$

$$C_{D_o} = -6.539E-10\ a^7 + 2.608E-8\ a^6 - 1.176E-7\ a^5 - 5.244E-6\ a^4 + 7.751E-5\ a^3 - 2.278E-5\ a^2 + 4.268E-5\ a + 0.0551$$

Side-Force-Due-to-Rolling Component

The side-force-due-to-rolling component, C_{y_p}, was computed using the following equation:

$$(C_{y_p})_{WBT} = (C_{y_p})_{WB} + 2((z-z_p)/b_w)(\Delta C_{y_B})_{v(WBT)} \qquad (2-63)$$

where $(C_{y_p})_{WBT}$ is the component for the entire wing-body-tail combination and $(C_{y_p})_{WB}$ is the component for the wing-body combination.

$(C_{y_p})_{WB}$ was calculated using the following equation:

$$(C_{y_p})_{WB} = K((C_{y_p}/C_L)C_L) + (\Delta C_{y_p})_{\Gamma} \qquad (2-64)$$

where

$$K = \frac{\frac{\partial}{\partial \alpha}(C_L \tan(\alpha)) - \frac{\partial}{\partial \alpha}(C_D - C_{D_o})}{\frac{\partial}{\partial \alpha}(C_L \tan(\alpha)) - \frac{\partial}{\partial \alpha}(C_L^2/\pi\ AR)} \qquad (2-65)$$

$$\left.\frac{C_{y_p}}{C_L}\right|_{\substack{C_L=0\\M}} = \frac{AR + 4\cos(\Lambda_{c/4})}{AR\ \beta + 4\cos(\Lambda_{c/4})} \qquad (2-66)$$

and

$$(\Delta C_{y_p})_{\Gamma} = (3\sin(\Gamma)\ (1-2\frac{z}{b/2}))\ (C_{l_p})_{\substack{\Gamma=0\\C_L=0}} \qquad (2-67)$$

Since C_L and C_D are functions of the flight condition, K was reduced to a polynomial function of angle of attack and is summarized below:

$$K = 4.722E{-}6\ a^7 - 1.83E{-}6\ a^6 + 4.235E{-}6\ a^5 + 5.206E{-}4\ a^4 - 0.00590\ a^3 + 0.007230\ a^2 + 0.0870\ a + 1.00$$

By assuming constant Mach number and using aircraft constants and charts available in <u>USAF</u> <u>DATCOM</u>, (C_{y_p}/C_L) and (C_{y_p}) reduced to:

$$(C_{y_p}/C_L) = -.06$$
$$(C_{y_p}) = .101$$

Yaw-Moment-Due-to-Roll Component

The yaw-moment-due-to-roll component, C_{n_p}, was computed using the following equation:

$$(C_{n_p})_{WBT} = (C_{n_p})_{WB} - (2/b_w)(l_p \cos(a) + z_p \sin(a))((z-z_p)/b_w)* (\Delta C_{y_B})_{V(WBH)} \qquad (2\text{-}69)$$

where $(C_{n_p})_{WBT}$ is the component for the wing-body-tail combination and $(C_{y_p})_{WB}$ is the component for the wing-body combination.

$(C_{n_p})_{WB}$ was calculated using the following equation:

$$(C_{n_p})_{WB} = -C_{l_p} \tan(a) - K(-C_{l_p} \tan(a) - (C_{n_p}/C_L)C_L) + (C_{n_p}/\theta)\theta \qquad (2\text{-}70)$$

where

$$\left.\frac{C_{n_p}}{C_L}\right|_{\substack{C_L=0\\M=0}} = -\frac{1}{6}\;\frac{AR + 6\,(AR + \cos(\Lambda_{c/4}))\left(\frac{\bar{x}}{\bar{c}}\,\frac{\tan(\Lambda_{c/4})}{AR} + \frac{\tan(\Lambda_{c/4})}{12}\right)}{AR + 4\cos(\Lambda_{c/4})} \qquad (2\text{-}71)$$

K was previously discussed, and C_{l_p} is the roll damping component to be discussed later. By assuming constant Mach number and using aircraft constants and charts available in <u>USAF DATCOM</u>, (C_{n_p}/C_L) and (C_{n_p}/θ) reduced to:

$$(C_{n_p}/C_L) = -.1003$$
$$(C_{n_p}/\theta) = 0.0$$

<u>Roll Damping Component</u>

The roll damping component, C_{l_p}, was computed using the following equation:

$$(C_{l_p})_{WBT} = (C_{l_p})_{WB} + .5(C_{l_p})_H(S_H/S_W)(b_H/b_W)^2 + 2(z/b_W) * ((z-z_p)/b_W)(\Delta C_{Y_p})_{V(WBT)} \qquad (2\text{-}72)$$

where $(C_{l_p})_{WBT}$ is the component for the wing-body-tail combination and $(C_{l_p})_{WB}$ is the component for the wing-body combination.

$(C_{l_p})_{WB}$ was calculated by the following equation:

$$(C_{l_p})_W = (\beta C_{l_p}/k)_{C_L=0}(k/\beta)(C_{L_\alpha})_{C_L}/(C_{L_\alpha})_{C_L=0}(C_{l_p})_\Gamma/(C_{l_p})_{\Gamma=0} + (\Delta C_{l_p})_{drag} \qquad (2\text{-}73)$$

where

$$(C_{l_p})_\Gamma/(C_{l_p})_{\Gamma=0} = 1 - 2(^z/(b/2))\sin(\Gamma) + 3(^z/(b/2))^2\sin^2(\Gamma) \quad (2\text{-}74)$$

$$(\Delta C_{l_p})_{drag} = (C_{l_p})_{C_{D_L}/C_L^2}{*}C_L^2 - (^1/8)C_{L_o} \quad (2\text{-}75)$$

$$\beta = \sqrt{1 - M^2} \quad (2\text{-}76)$$

$$k = (C_{l_a})_M/(2\pi/\beta) \quad (2\text{-}77)$$

$$(C_{l_a})_M = (1.05/\beta)(C_{l_a}/(C_{l_a})_{theory})(C_{l_a})_{theory} \quad (2\text{-}78)$$

Assuming constant Mach number and using aircraft constants and charts available in <u>USAF</u> <u>DATCOM</u>, $(C_{l_p})_\Gamma/(C_{l_p})_{\Gamma=0}$, β, k, and $(\beta C_{l_p}/k)$ reduced to:

$$(C_{l_p})_\Gamma/(C_{l_p})_{\Gamma=0} = .985$$

$$\beta = .990$$

$$k = .992$$

$$(\beta C_{l_p}/k) = -.42$$

Since both $(\Delta C_{l_p})_{drag}$ and (C_{l_a}/C_{l_a}) are functions of the lift and drag coefficients, it was possible to reduce the equations to a set of polynomial equations in angle of attack, summarized below:

$$\frac{(C_{L_a})_{C_L}}{(C_{L_a})_{C_L=0}} = -4.844E\text{-}10\ a^7 + 1.797E\text{-}7\ a^6 - 6.524E\text{-}6\ a^5 + 6.049E\text{-}5\ a^4 - 2.235E\text{-}5\ a^3 - 1.326E\text{-}3\ a^2 - 1.720E\text{-}3\ a + 0.99$$

$$\Delta C_{l_{p_1}} = 8.453E\text{-}10\ a^7 - 5.066E\text{-}8\ a^6 + 1.000E\text{-}6\ a^5 - 9.040E\text{-}6\ a^4 + 1.985E\text{-}5\ a^3 + 2.109E\text{-}4\ a^2 + 9.369E\text{-}5\ a + 0.1329$$

$$\Delta C_{l_{p_o}} = 2.325E\text{-}11\ a^7 - 5.324E\text{-}10\ a^6 - 1.689E\text{-}8\ a^5 + 4.544E\text{-}7\ a^4 - 3.006E\text{-}6\ a^3 + 5.360E\text{-}5\ a^2 + 2.386E\text{-}4\ a - 0.00675$$

Side-Force-Due-to-Yaw Component

The USAF DATCOM lists no method for the side-force-due-to-yaw component. In addition, it states that the term is usually negligible for angles of attack up to stall. Therefore, this term was neglected for this study.

Yaw Damping Component

The yaw damping component, C_{n_r}, was computed using the following equation:

$$(C_{n_r})_{WBT} = (C_{n_r})_{WB} + (^2/b_w^2)(l_p\cos(a) + z_p\sin(a))^2(\Delta C_{Y_B})_{V(WBT)} \qquad (2\text{-}79)$$

where $(C_{n_r})_{WBT}$ is the component for the wing-body-tail combination and $(C_{n_r})_{WB}$ is the component for the wing-body combination.

$(C_{n_r})_{WB}$ was calculated using the following equation:

$$(C_{n_r})_{WB} = (^C n_r/C_L^2)C_L^2 + (^C n_r/C_{D_o})C_{D_o} \qquad (2\text{-}80)$$

where

$$C_{D_o} = C_D - C_L^2/(\pi AR) \qquad (2\text{-}81)$$

Assuming constant Mach number and using aircraft constants and charts available in **USAF DATCOM**, $({}^{C_{n_r}}/C_L^2)$ and $({}^{C_{n_r}}/C_{D_o})$ reduced to:

$$({}^{C_{n_r}}/C_L^2) = -.02$$
$$({}^{C_{n_r}}/C_{D_o}) = -.32$$

Roll-Moment-Due-to-Yaw Component

The roll-moment-due-to-yaw component, C_{l_r}, was calculated using the following equation:

$$(C_{l_r})_{WBT} = (C_{l_r})_{WB} - ({}^{2}/b_w^2)(l_p\cos(\alpha) + z_p\sin(\alpha)) * (z_p\cos(\alpha) - l_p\sin(\alpha))(\Delta C_{y_\beta})_{V(WBH)} \qquad (2\text{-}82)$$

where $(C_{l_r})_{WBT}$ is the component for the wing-body-tail combination and $(C_{l_r})_{WB}$ is the component for the wing-body combination.

$(C_{l_r})_{WB}$ was computed by the following equation:

$$(C_{l_r})_{WB} = ({}^{C_{l_r}}/C_L)_{C_L=0}C_L + (C_{l_r})_{C_L}C_L + ({}^{\Delta C_{l_r}}/\Gamma)\Gamma \qquad (2\text{-}83)$$

where

$$\left.\frac{C_{l_r}}{C_L}\right|_{\substack{C_L=0\\M}} = \frac{1 + \frac{AR\,(1 - B^2)}{2\,(AR\,B + 2\cos(\Lambda_{c/4}))} + \frac{AR + 2\cos(\Lambda_{c/4})}{AR\,B + 4\cos(\Lambda_{c/4})}\,\frac{\tan^2(\Lambda_{c/4})}{8}}{1 + \frac{AR + 2\cos(\Lambda_{c/4})}{AR + 4\cos(\Lambda_{c/4})}\,\frac{\tan^2(\Lambda_{c/4})}{8}}\;\left.\frac{C_{l_r}}{C_L}\right|_{\substack{C_L=0\\M=0}} \qquad (2\text{-}84)$$

$$\frac{\Delta C_{l_r}}{\Gamma} = \frac{1}{12}\;\frac{\pi * AR\sin(\Lambda_{c/4})}{AR + 4*\cos(\Lambda_{c/4})} \qquad (2\text{-}85)$$

Assuming constant Mach number and using aircraft constants and charts available in <u>USAF DATCOM</u>, these terms reduced to:

$$(C_{l_r}/C_L) = .241$$
$$(\Delta C_{l_r}/\Gamma) = .001079$$

2.3.4 Control Derivative Components

As in the cases of the constant and stability derivative components, static data were available for the control derivatives. Because the data were not presented in the same way, rudder and aileron are discussed separately.

Rudder Derivative Components

The method for finding the rudder derivative component equations was the same as the stability derivative components. Static data were presented in one plot per throttle setting with five curves per plot. Each curve (corresponding to a different value of rudder deflection -- -17.5, -9.0, 0.0, 7.0, and 15.2 deg) showed how the coefficients varied with angle of attack. Once again, eight points per curve were selected, and they are presented in Appendix A.

The first task was to compute the rudder derivatives at each angle of attack and throttle setting using a linear regression. The linear assumption was valid for both C_y and C_n but not so good for C_l. Since it was desirable to use as simple a model as possible, linearity for C_l was assumed, although the assumption could be reconsidered if the results were not satisfactory. The rudder derivatives then were reduced to the form:

$$C_{y_{dR}} = C_{y_{dR_T}} T_c + C_{y_{dR_0}} \qquad (2\text{-}86)$$

$$C_{n_{dR}} = C_{n_{dR_T}} T_c + C_{n_{dR_0}} \qquad (2\text{-}87)$$

$$C_{l_{dR}} = C_{l_{dR_T}} T_c + C_{l_{dR_0}} \qquad (2\text{-}88)$$

Since only two throttle points were used, no investigation of the linearity of throttle was necessary. A linear regression was also used for this task. The slope and constant terms were then curve fitted using the method previously discussed. The final reduced equations are summarized below:

$$C_{y_{dR_T}} = 6.225E\text{-}10\ a^7 - 4.079E\text{-}8\ a^6 + 9.409E\text{-}7\ a^5 - 8.651E\text{-}6\ a^4 + 1.617E\text{-}5\ a^3 + 1.582E\text{-}4\ a^2 - 2.147E\text{-}4\ a + 0.0088$$

$$C_{y_{dR_0}} = 4.687E\text{-}11\ a^7 - 3.079E\text{-}9\ a^6 - 7.235E\text{-}8\ a^5 - 6.912E\text{-}7\ a^4 + 1.695E\text{-}6\ a^3 + 6.535E\text{-}6\ a^2 - 1.206E\text{-}5\ a + 0.00296$$

$$C_{n_{dR_T}} = -8.[illegible]38E\text{-}11\ a^7 + 4.476E\text{-}9\ a^6 - 7.650E\text{-}8\ a^5 + 5.046E\text{-}7\ a^4 - 1.104E\text{-}6\ a^3 - 4.531E\text{-}6\ a^2 - 8.116E\text{-}5\ a - 0.0043$$

$$C_{n_{dR_0}} = -7.266E\text{-}11\ a^7 + 4.832E\text{-}9\ a^6 - 1.141E\text{-}7\ a^5 + 1.077E\text{-}6\ a^4 - 2.106E\text{-}6\ a^3 - 1.690E\text{-}5\ a^2 + 2.946E\text{-}5\ a - 0.00156$$

$$C_{l_{dR_T}} = 3.278E\text{-}10\ a^7 - 2.428E\text{-}8\ a^6 + 6.533E\text{-}7\ a^5 - 7.240E\text{-}6\ a^4 + 2.069E\text{-}5\ a^3 + 1.294E\text{-}4\ a^2 - 6.396E\text{-}4\ a + 0.00188$$

$$C_{l_{dR_0}} = -1.381E\text{-}10\ a^7 + 9.568E\text{-}9\ a^6 - 2.394E\text{-}7\ a^5 + 2.467E\text{-}6\ a^4 -$$

$$6.880E-6\ a^{3} - 4.086E-5\ a^{2} + 1.717E-4\ a + 0.000247$$

Aileron Derivative Components

The only difference between the aileron derivatives and the rudder derivatives was that the aileron derivatives were presented in **NASA TN D-5857** with only one throttle setting; hence, one step in the process was deleted. The data were presented as five curves per plot (corresponding to aileron values of -42, -21, 0, 21, and 42 deg) showing the effect of angle of attack and are summarized in **Appendix A**. The curve fitting method previously discussed was used to reduce the data to a final set of equations summarized as follows:

$$C_{y_{dA}} = 2.433E-11\ a^{7} - 1.603E-9\ a^{6} + 3.859E-8\ a^{5} - 3.817E-7\ a^{4} + 8.667E-7\ a^{3} + 5.518E-6\ a^{2} - 2.085E-5\ a - 0.000267$$

$$C_{n_{dA}} = -1.037E-11\ a^{7} + 1.088E-9\ a^{6} - 2.596E-8\ a^{5} + 2.497E-7\ a^{4} - 5.133E-7\ a^{3} - 4.023E-6\ a^{2} + 3.243E-5\ a - 0.000005$$

$$C_{l_{dA}} = -1.361E-11\ a^{7} + 9.800E-10\ a^{6} - 2.405E-8\ a^{5} + 2.227E-7\ a^{4} - 2.285E-7\ a^{3} - 5.377E-6\ a^{2} + 3.619E-6\ a - 0.00127$$

2.4 OPEN-LOOP RESULTS

The final step in the model development was verification of the model. To do this, a number of test flight conditions were used to determine the linearized system dynamic equations, F and G. From these, system eigenvalues and dimensional derivatives were determined. Table 1 lists the results.

Twenty-seven flight condition combinations were used representing three different values for each of the three flight condition variables. The three values were chosen as the maximum, minimum, and midpoint values for each of the variable's typical ranges. In particular, the values for angle of attack were -4, 10, and 24 deg; for throttle setting, they were .05, .15, and .25; and for dynamic pressure, they were 9.731, 21.894, and 38.922 pounds per square foot (corresponding to velocities of 100, 150, and 200 feet per second).

The eigenvalues give a good representation of the basic aircraft. At low angles of attack (-4°), no instabilities are noted. At moderate angles of attack (10°), an unstable spiral mode with a long time constant is encountered. Finally, at high angles of attack (24°), an unstable roll-spiral develops. This corresponds to the wing rock instability which has been noted in this flight condition regime.

In addition, the rotary derivatives were checked against experimental data in **NASA TN D-6643**. The comparison was done using for the model at a nominal condition (angle of attack = 10°; throttle =.13; velocity = 150 ft/sec). The yaw damping (C_{n_r}) and the roll damping (C_{l_p}) terms correspond well with accepted values. The yaw damping for the model was -0.125 compared with -0.125 for the data, while the roll damping was -0.370 compared with -0.41. The cross-coupling derivatives, however, both were off by a factor of two. Yaw-due-to-roll (C_{n_p}) was -0.110 for the model compared to -0.056 in the data, while roll-due-to-yaw (C_{l_r}) was 0.236 compared to 0.107. Since no error could be found in the calculations and since the complete model appeared to give good results despite this discrepancy, the cross-coupling derivatives were left as derived. This assumption could be reconsidered if later results were not satisfactory.

2.5 MODEL REEVALUATION

After this project was finished, an inspection of the model was done and two major errors were found along with other problems noted in later Chapters, these errors may prove to be the reason the control system was not a complete success. This section has been added after the fact to point out these problems and is included in this chapter to be consistent with the thesis organization.

Both model errors pertain to assumptions that were made which are not valid at high angles of attack. The first involved the inertia matrix; the second involved the vertical component of the velocity vector. The model errors in turn affect the control law and the eventual outcome of the project.

2.5.1 Inertia Matrix

Due to the emphasis on angle of attack that this project used, the stability axes provided a better basis for an axis system than did the body axes. Indeed, much of the aerodynamic data that was used to formulate the model was based on the stability-axis system. The body-axis equations of motion were rotated through the angle of attack for development of the control law. In doing so, however, the rotation effects on the inertia matrix were not considered. Hence, the model essentially used stability-axis aerodynamics and body-axis inertias. At low angles of attack (less than 5 degrees), this effect can be considered negligible. However, this control system was required to perform at high angles of attack and the inertia changes are significant.

To repair this oversight, the inertia matrix needs to be rotated to the stability axis system and can be done by the following equation:

$$I_S = H_B^S I_B H_S^B$$

I_B is defined as the body-, or principle-axis inertia matrix as follows:

$$I_B = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}$$

H_B^S and H_S^B are defined as follows:

$$H_B^S = \begin{bmatrix} \cos\alpha_o & 0 & \sin\alpha_o \\ 0 & 1 & 0 \\ -\sin\alpha_o & 0 & \cos\alpha_o \end{bmatrix}$$

$$H_S^B = H_B^{S^{-1}} = H_B^{S^T}$$

By working through the equation, the following relationships result:

$$I_{xx_S} = I_x \cos^2\alpha_o + I_z \sin^2\alpha_o$$

$$I_{yy_S} = I_y$$

$$I_{zz_S} = I_x \sin^2\alpha_o + I_z \cos^2\alpha_o$$

$$I_{xz_S} = \frac{1}{2}(I_z - I_x)\sin 2\alpha_o$$

and the stability-axis inertia matrix is as follows:

$$I_S = \begin{bmatrix} I_{xx_S} & 0 & -I_{xz_S} \\ 0 & I_{yy_S} & 0 \\ -I_{xz_S} & 0 & I_{zz_S} \end{bmatrix}$$

Perhaps the most significant effect of this rotation is the appearance of the product-of-inertia term, I_{xz_S}. This term affects the rotational dynamics of the aircraft.

The rotational dynamics are defined as follows:

$$\dot{\underline{\omega}}_S = I_S^{-1}[\underline{M}_S - \tilde{\underline{\omega}}_S I_S \underline{\omega}_S]$$

where the terms are as follows:

$$\underline{M}_S = \begin{bmatrix} C_\ell \bar{q}Sb \\ C_m \bar{q}Sc \\ C_n \bar{q}Sb \end{bmatrix}_S$$

$$\tilde{\underline{\omega}}_S = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}_S$$

$$\underline{\omega}_S = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_S$$

Working through the equations and assuming pitch rate (q) is negligible, the following equations result:

$$\dot{p}_S = \left(\frac{1}{I_{xx_S} I_{zz_S} - I_{xz_S}^2}\right) [C_\ell \bar{q}SbI_{zz_S} + C_n \bar{q}SbI_{xz_S}]$$

$$\dot{r}_S = \left(\frac{1}{I_{xx_S} I_{zz_S} - I_{xz_S}^2}\right) [C_\ell \bar{q} S b I_{xz_S} + C_n \bar{q} S b I_{xx_S}]$$

Comparing these equations to (2-7) and (2-9), the inertia effects are readily apparent: the higher the angle of attack, the larger the inertia effects.

2.5.2 Vertical Component of Velocity

The rate of change of the lateral velocity component, $\dot{v}$, and hence the rate of change of sideslip, $\dot{\beta}$, are functions of the vertical component of velocity, w. At low angles of attack, w is very small compared to u, the forward velocity component, but at high angles of attack, w takes on significance. This term, however, was inadvertently neglected (assumed zero) and hence does not appear in equation (2-8) for $\dot{\beta}$.

The result of this oversight is the miscalculation of the F matrix. The Δp coefficient of the $\Delta\dot{\beta}$ equation changes from Y_p/V_o to $Y_p/V_o + w/V_o$. At high angles of attack (e.g. 24 degrees), the coefficient becomes

$$Y_p/V_o + w/V_o = 0.4$$

instead of

$$Y_p/V_o = 0.008$$

The significance of this error, then, is readily apparent.

Table 1

Open-Loop Results

FLIGHT CONDITION			EIGENVALUES	STABILITY DERIVATIVES			ROTARY DERIVATIVES						CONTROL DERIVATIVES					
α	T_c	$\bar{q}$		N_β	Y_β/V_o	L_β	N_r	Y_r/V_o	L_r	N_p	Y_p/V_o	L_p	$N_{\delta R}$	$Y_{\delta R}/V_o$	$L_{\delta R}$	$N_{\delta A}$	$Y_{\delta A}/V_o$	$L_{\delta A}$
-4	.03	9.731	-.1782 ± 1.5991j -3.3327 -.0741	2.363	-.157	-4.816	-.330	0	.170	.117	.005	-3.277	-1.601	.033	.908	-.075	-.003	- 3.387
-4	.03	21.894	-.3436 ± 2.3074j -4.9038 - .0539	5.316	-.235	-10.836	-.494	0	-.255	.176	.005	-4.916	-3.602	.049	2.043	-.169	-.004	- 7.620
-4	.03	38.922	-.4964 ± 3.0328j -6.4921 - .0417	9.450	-.313	-19.264	-.659	0	-.340	.234	.005	-6.554	-6.404	.065	3.632	-.301	-.007	-13.547
-4	.13	9.731	-.1798 ± 1.6123j -3.2458 - .0765	2.369	-.177	- 5.224	-.329	0	-.132	.108	.005	-3.175	-1.998	.039	1.535	-.075	-.003	- 3.387
-4	.13	21.894	-.3534 ± 2.3178j -4.7598 - .0562	5.330	-.265	-11.754	-.494	0	-.197	.162	.005	-4.763	-4.496	.059	3.453	-.169	-.005	- 7.620
-4	.13	38.922	-.5136 ± 3.0419j -6.2931 - .0437	9.476	-.354	-20.895	-.659	0	-.263	.217	.005	-6.351	-7.992	.078	6.139	-.301	-.007	-13.547
-4	.23	9.731	-.1804 ± 1.6262j -3.1606 - .0789	2.375	-.197	- 5.632	-.329	0	-.093	.100	.005	-3.074	-2.395	.046	2.161	-.075	-.003	- 3.387
-4	.23	21.894	-.3623 ± 2.3292j -4.6174 - .0585	5.344	-.296	-12.671	-.494	0	-.139	.149	.005	-4.610	-5.389	.068	4.863	-.169	-.005	- 7.260
-4	.13	38.922	-.5298 ± 3.0524j -6.0956 - .0456	9.501	-.395	-22.527	-.659	0	-.186	.199	.005	-6.147	-9.580	.091	8.645	-.301	-.007	-13.547
10	.03	9.731	-.2177 ± 1.5760j -3.1487j - .0406	1.608	-.122	- 4.320	-.390	0	1.674	-.312	.006	-3.032	-1.869	.036	.350	.242	-.004	- 3.563
10	.03	21.894	-.3691 ± 2.2532j -4.6074 .0303	3.619	-.182	- 9.720	- .586	0	2.511	-.467	.006	-4.547	-4.205	.054	.788	.545	-.006	- 8.017
10	.03	38.922	-.5125 ± 2.9488j -6.0859 .0237	6.433	-.243	-17.280	- .781	0	3.349	-.623	.006	-6.063	-7.476	.071	1.400	.970	-.008	-14.252
10	.13	9.731	-.2516 ± 1.6844j -3.0235 .0691	1.897	-.149	- 4.074	- .392	0	1.835	-.33	.006	-2.916	-2.467	.047	.495	.242	-.004	- 3.563
10	.13	21.894	-.4124 ± 2.4206j -4.4127 .0513	4.268	-.224	- 9.167	- .588	0	2.753	-.506	.006	-4.374	-5.551	.070	1.113	.545	-.006	- 8.017
10	.13	38.922	-.5663 ± 3.1742j -5.8225 .0400	7.588	-.299	-16.296	- .784	0	3.671	-.675	.006	-5.832	-9.868	.093	1.978	.970	-.008	-14.252

Table 1

continued

FLIGHT CONDITION			EIGENVALUES	STABILITY DERIVATIVES			ROTARY DERIVATIVES						CONTROL DERIVATIVES					
α	T_c	$\bar{q}$		N_β	Y_β/V_o	L_β	N_r	Y_r/V_o	L_r	N_p	Y_p/V_o	L_p	$N_{\delta R}$	$Y_{\delta R}/V_o$	$L_{\delta R}$	$N_{\delta A}$	$Y_{\delta A}/V_o$	$L_{\delta A}$
10	.23	9.731	-.2851 ± 1.7894j -2.8981 .0965	2.186	-.177	- 3.828	- .394	0	1.996	-.364	.006	-2.801	-3.065	.057	.639	.242	-.004	- 3.563
10	.23	21.894	-.4564 ± 2.5809j -4.2162 .0713	4.917	-.266	- 8.613	- .591	0	2.994	-.545	.006	-4.201	-6.897	.086	1.438	.545	-.006	- 8.017
10	.23	38.922	-.6217 ± 3.3894j -5.5558 .0557	8.742	-.354	-15.313	- .787	0	3.993	-.727	.006	-5.602	-12.261	.115	2.556	.970	-.008	-14.252
24	.03	9.731	-.9436 ± .8959j .3567 ± 6514j	1.400	-.080	- 8.616	- .714	0	2.321	-.399	.008	- .380	- .867	.035	-.035	.828	.003	- 2.965
24	.03	21.894	-1.5151 ± 1.7827j .7609 .5050	3.149	-.120	-19.385	-1.071	0	3.481	.599	.008	- .569	-1.950	.053	-1.864	.797	.005	- 6.672
24	.03	38.922	-2.0765 ± 2.5363j 1.5868 .2191	5.599	-.160	-34.462	-1.428	0	4.641	.799	.008	-.759	-3.466	.070	-3.313	1.416	.006	-11.860
24	.13	9.731	- .8778 ± .7933j .3502 ± .5483j	1.279	-.094	- 7.139	- .718	0	2.564	.391	.008	-.244	-1.085	.033	- .743	.354	.003	- 2.965
24	.13	21.894	-1.4257 ± 1.6231j .9757 .2928	2.877	-.140	-16.062	-1.076	0	3.847	.586	.008	-.366	-2.441	.050	-1.673	.797	.005	- 6.672
24	.13	38.922	-1.9604 ± 2.3177j 1.6548 .1555	5.115	-.187	-28.555	-1.435	0	5.129	.782	.008	-.488	-4.339	.066	-2.973	1.416	.006	-11.860
24	.23	9.731	- .7944 ± .6651j .3258 ± .3785j	1.158	-.107	- 5.662	- .721	0	2.808	.382	.008	-.109	-1.303	.031	- .658	.354	.003	- 2.965
24	.23	21.894	-1.3331 ± 1.4401j 1.1209 .1395	2.605	-.161	-12.739	-1.082	0	4.212	.573	.008	-.163	-2.932	.047	-1.482	.797	.005	- 6.672
24	.23	38.922	-1.8413 ± 2.0684j 1.7271 .0808	4.632	-.214	-22.648	-1.443	0	5.617	.765	.008	-.217	-5.212	.062	-2.634	1.416	.006	-11.860

Chapter III

DEVELOPMENT OF THE CONTROL LAW

The design of the command/stability augmentation system was accomplished using linear-quadratic control theory for a sampled data regulator. This method calculates an optimal feedback matrix, C, by minimizing a sampled-data cost function, J, specified by:

$$J = 1/2\sum_{k=0}^{\infty}\int_{t_k}^{t_{k+1}} (\Delta\underline{x}^T Q_c \Delta\underline{x} + \Delta\underline{u}^T R_c \Delta\underline{u})\ dt \qquad (3\text{-}1)$$

(where Q_c and R_c are the continuous-time state and control weighting matrices, respectively) subject to a linear constraint specified by the sampled-data system dynamic equations:

$$\Delta\underline{x}_{k+1} = \Phi\ \Delta\underline{x}_k + \Gamma\Delta\underline{u}_k \qquad (3\text{-}2)$$

where Φ and Γ are the sampled-data equivalents of the system dynamics equations, F and G.

A non-zero set point regulator was formulated as follows:

$$\Delta\underline{u} = \Delta\underline{u}^* - C\ \ (\Delta\underline{x} - \Delta\underline{x}^*) \qquad (3\text{-}3)$$

where $\Delta\underline{x}^*$ and $\Delta\underline{u}^*$ are the equilibrium set points for the states and controls determined by the command, $\Delta\underline{y}^*$, and where $\Delta\underline{x}$ and $\Delta\underline{u}$ are the current values of the states and controls. The objective of this regulator, then, is to drive the states and controls to their equilibrium values.

The controller design task was two-fold. First, the equilibrium values of the states and controls were determined given a specified command input. Since the state includes an integral of a command vector element, the singular command equilibrium method was used. The second task was determination of the optimal feedback gain matrix for a sampled-data regulator, G. Once these tasks were completed, the control law specified by (3-3) was developed.

Since the CAS was designed as a sampled-data controller, a sampling time had to be selected. The criterion for the selection was that the sampling time had to be long enough to enable all calculations for the control law to be completed and short enough that aircraft handling was not degraded. A sampling time of 0.1 seconds appeared to make a good compromise between the two conflicting objectives.

This chapter covers the determination of the singular command equilibrium, the calculation of the optimal gains, and the development of the control law. In addition, results are included showing: the selection process of the continuous-time matrices, Q_c and R_c; a detailed description of the controller operation for a nominal flight condition, with simulation results; and a summary of closed-loop simulation results for 27 flight conditions.

3.1 SINGULAR COMMAND EQUILIBRIUM

The first step toward the design of the controller is the determination of the equilibrium point. The equilibrium point is defined as the desired value of the aircraft's states plus the assorted control settings. It is determined by the input command vector, $\Delta\underline{y}^*$, and it is represented by $\Delta\underline{x}^*$ and $\Delta\underline{u}^*$, the equilibrium values of the states and controls.

The system equations are denoted by:

$$\Delta\dot{\underline{x}} = F\ \Delta\underline{x} + G\ \Delta\underline{u} \qquad (3-4)$$

$$\Delta\underline{y} = H_x\ \Delta\underline{x} + H_u\ \Delta\underline{u} \qquad (3-5)$$

To examine the equilibrium, it would seem correct to set $\Delta\underline{x} = \underline{0}$ and manipulate (3-4) and (3-5) as follows:

$$\begin{bmatrix} 0 \\ \Delta\underline{y} \end{bmatrix} = \begin{bmatrix} F & G \\ H_x & H_u \end{bmatrix} \begin{bmatrix} \Delta\underline{x} \\ \Delta\underline{u} \end{bmatrix} \qquad (3-6)$$

$$\begin{bmatrix} \Delta\underline{x}^* \\ \Delta\underline{u}^* \end{bmatrix} = \begin{bmatrix} \Delta\underline{x} \\ \Delta\underline{u} \end{bmatrix} = \begin{bmatrix} F & G \\ H_x & H_u \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \Delta\underline{y}^* \end{bmatrix} \qquad (3-7)$$

(where $\Delta\underline{y}^* = \Delta\underline{y}$).

However, in the case of a singular command, $\begin{bmatrix} F & G \\ H_x & H_u \end{bmatrix}^{-1}$ does not exist, and a different approach (Ref.6) must be taken.

Singular equilibrium occurs when the state vector, $\Delta \underline{x}$, contains an integral of a command vector ($\Delta \underline{y}$) element. While such a case has desirable aspects, it does mean that no true equilibrium can exist in the states and controls. A singular command variable (in this case, roll rate) is the derivative of a state variable (roll angle), so a non-zero value of the former prevents the latter from reaching any steady-state value (hence $\underline{x}_2^* \neq \underline{0}$). However, the disequilibrium in the singular variable may affect the nonsingular variables such that they, too, do not reach steady-state. While the disequilibrium in the nonsingular variables is small, it is still significant enough to affect the results and should not be neglected. Indeed, singular equilibrium implies no equilibrium at all (Ref. 5).

To develop the singular equilibrium, the singular and nonsingular variables are partitioned, resulting in the following equations:

$$\Delta \underline{x} = \begin{bmatrix} \Delta \underline{x}_1 \\ \Delta \underline{x}_2 \end{bmatrix} = \begin{bmatrix} \text{nonsingular variables} \\ \text{singular variables} \end{bmatrix}$$

$$\begin{bmatrix} \Delta \underline{x}_1 \\ \Delta \underline{x}_2 \end{bmatrix} = \begin{bmatrix} F_1 & F_{12} \\ F_{21} & F_2 \end{bmatrix} \begin{bmatrix} \Delta \underline{x}_1 \\ \Delta \underline{x}_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \Delta \underline{u} \qquad (3-8)$$

$$\Delta \underline{y} = \begin{bmatrix} H_{x_1} & H_{x_2} \end{bmatrix} \begin{bmatrix} \Delta \underline{x}_1 \\ \Delta \underline{x}_2 \end{bmatrix} + H_u \, \Delta \underline{u} \qquad (3-9)$$

Thus, the following equations are applicable for finding the equilibrium values:

$$\Delta \dot{\underline{x}}_1^* = F_1 \Delta \underline{x}_1^* + F_{12}\Delta \underline{x}_2^* + G_1 \Delta \underline{u}^* \qquad (3\text{-}10)$$

$$\Delta \dot{\underline{x}}_2^* = F_{21}\Delta \underline{x}_1^* + F_2 \Delta \underline{x}_2^* + G_2 \Delta \underline{u}^* \qquad (3\text{-}11)$$

$$\Delta \underline{y}^* = H_{x_1}\Delta \underline{x}_1^* + H_{x_2}\Delta \underline{x}_2^* + H_u \Delta \underline{u}^* \qquad (3\text{-}12)$$

and the following values are assumed:

$$\Delta \underline{x}_2^*(0) = \underline{0}$$

$$\Delta \underline{y}^* = \text{constant}$$

$$\Delta \dot{\underline{x}}_2^* = K\Delta \underline{y}^* \quad (\text{thus } \Delta \dot{\underline{x}}_2^* = \text{constant})$$

Solving for $\underline{x}_1^*$ and $\underline{u}^*$ in (3-10) and (3-12), the following matrix equation results:

$$\begin{bmatrix} \Delta \underline{x}_1^* \\ \Delta \underline{u}^* \end{bmatrix} = \begin{bmatrix} F_1 & G_1 \\ H_{x_1} & H_u \end{bmatrix}^{-1} \begin{bmatrix} \Delta \dot{\underline{x}}_1^* - F_{12}{}^*\Delta \underline{x}_2^* \\ \Delta \underline{y}^* - H_{x_2}{}^*\Delta \underline{x}_2^* \end{bmatrix} \qquad (3\text{-}13)$$

S can be defined as,

$$\begin{bmatrix} F_1 & G_1 \\ H_{x_1} & H_u \end{bmatrix}^{-1} = S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

where the partitions of S can alternately be found by:

$$S_{11} = F_1^{-1}(-G_1 S_{21} + I) \qquad (3\text{-}14)$$

$$S_{12} = -F_1^{-1}G_1 S_{22} \qquad (3\text{-}15)$$

$$S_{21} = -S_{22}H_{x_1}F_1^{-1} \qquad (3\text{-}16)$$

$$S_{22} = (-H_{x_1}F_1^{-1}G_1 + H_u)^{-1} \qquad (3\text{-}17)$$

Thus, multiplied out, equation (3-13) becomes:

$$\Delta \underline{x}_1^* = S_{12}\Delta \underline{y}^* - (S_{11}F_{12} + S_{12}H_{x_2})\Delta \underline{x}_2^* + S_{11}\Delta \dot{\underline{x}}_1^* \qquad (3\text{-}18)$$

$$\Delta\underline{u}^* = S_{22}\Delta\underline{y}^* - (S_{21}F_{12} + S_{22}H_{x_2})\Delta\underline{x}_2^* + S_{21}\Delta\dot{\underline{x}}_1^* \qquad (3\text{-}19)$$

Substituting (3-18) and (3-19) into (3-11) and gathering terms, the following equation results:

$$\Delta\dot{\underline{x}}_2^* = (F_{21}S_{12} + G_2S_{22})\Delta\underline{y}^* + (F_{21}S_{11} + G_2S_{21})\Delta\dot{\underline{x}}_1^*$$
$$(F_2 - F_{21}(S_{11}F_{12} + S_{12}H_{x_2}) - G_2(S_{21}F_{12} + S_{22}H_{x_2}))\Delta\underline{x}_2^* \qquad (3\text{-}20)$$

Equation (3-20) can be simplified by making the following definitions:

$$K_y = F_{21}S_{12} + G_2S_{22} \qquad (3\text{-}21)$$

$$K_{\dot{x}} = F_{21}S_{11} + G_2S_{21} \qquad (3\text{-}22)$$

$$K_x = F_2 - F_{21}(S_{11}F_{12} + S_{12}H_{x_2}) - G_2(S_{21}F_{12} + S_{22}H_{x_2}) \qquad (3\text{-}23)$$

such that:

$$\Delta\dot{\underline{x}}_2^* = K_y\Delta\underline{y}^* + K_{\dot{x}}\Delta\dot{\underline{x}}_1^* + K_x\Delta\underline{x}_2^* \qquad (3\text{-}24)$$

Recalling the assumption that $\Delta\dot{\underline{x}}_2^* = K\Delta\underline{y}^*$:

$$\Delta\dot{\underline{x}}_2^* = K\,\Delta\underline{y}^*$$
$$= K\begin{bmatrix} H_{x_1} & H_{x_2} \end{bmatrix}\begin{bmatrix} \Delta\underline{x}_1^* \\ \Delta\underline{x}_2^* \end{bmatrix} + KH_u\Delta\underline{u}^*$$
$$= KH_{x_1}\Delta\underline{x}_1^* + KH_{x_2}\Delta\underline{x}_2^* + KH_u\Delta\underline{u}^* \qquad (3\text{-}25)$$

The following relationships are noted by comparing equation (3-25) to (3-11):

$$F_{21} = KH_{x_1} \qquad (3\text{-}26)$$

$$F_2 = KH_{x_2} \qquad (3\text{-}27)$$

$$G_2 = KH_u \qquad (3\text{-}28)$$

These relationships coupled with equations (3-14) to (3-17) allowed (3-24) to be simplified. First, $K_{\dot{x}}$ is eliminated by manipulating (3-22):

$$\begin{aligned}
K_{\dot{x}} &= F_{21}S_{12} + G_2S_{22} \\
&= KH_{x_1}F_1^{-1}(-G_1S_{21} + I) + KH_uS_{21} \\
&= KH_{x_1}F_1^{-1} - KH_{x_1}F_1^{-1}G_1S_{21} + KH_uS_{21} \\
&= K(H_{x_1}F_1^{-1} + (-H_{x_1}F_1^{-1}G_1 + H_u)S_{21}) \\
&= K(H_{x_1}F_1^{-1} + S_{22}^{-1}(-S_{22}H_{x_1}F_1^{-1})) \\
&= K(H_{x_1}F_1^{-1} - H_{x_1}F_1^{-1}) \\
&= 0
\end{aligned}$$

By the same reasoning, K_x is eliminated by manipulating (3-23):

$$\begin{aligned}
K_x &= F_2 - F_{21}(S_{11}F_{12} + S_{12}H_{x_2}) - G_2(S_{21}F_{12} + S_{22}H_{x_2}) \\
&= KH_{x_2} - KH_{x_1}S_{11}F_{12} - KH_{x_1}S_{12}H_{x_2} - KH_uS_{21}F_{12} - KH_uS_{22}H_{x_2} \\
&= KH_{x_2} - KH_{x_1}F_1^{-1}(-G_1(-S_{22}H_{x_1}F_1^{-1}) + I)F_{12} - \\
&\qquad KH_{x_1}(-F_1^{-1}G_1S_{22})H_{x_2} - KH_u(-S_{22}H_{x_1}F_1^{-1})F_{12} - KH_uS_{22}H_{x_2} \\
&= KH_{x_2} - KH_{x_1}F_1^{-1}G_1S_{22}(H_{x_1}F_1^{-1}F_{12} - H_{x_2}) - KH_{x_1}F_1^{-1}F_{12} \\
&\qquad KH_uS_{22}(H_{x_1}F_1^{-1}F_{12} - H_{x_2}) \\
&= KH_{x_2} + (KH_u - KH_{x_1}F_1^{-1}G_1)S_{22}(H_{x_1}F_1^{-1}F_{12} - H_{x_2}) - KH_{x_1}F_1^{-1}F_{12} \\
&= KH_{x_2} + K(H_u - H_{x_1}F_1^{-1}G_1)(-H_{x_1}F_1^{-1}G_1 + H_u)^{-1}* \\
&\qquad (H_{x_1}F_1^{-1}F_{12} - H_{x_2}) - KH_{x_1}F_1^{-1}F_{12} \\
&= KH_{x_2} + K(H_{x_1}F_1^{-1}F_{12} - H_{x_2}) - KH_{x_1}F_1^{-1}F_{12} \\
&= 0
\end{aligned}$$

Thus, $\Delta\dot{\underline{x}}_2^*$ reduced to:

$$\begin{aligned}\Delta\dot{\underline{x}}_2^{*} &= K_y\Delta\underline{y}^{*} + \cancel{K_x}\overset{0}{\Delta\underline{x}_2^{*}} + K_{\dot{x}}\overset{0}{\Delta\dot{\underline{x}}_1^{*}}\\ &= K_y\Delta\underline{y}^{*}\end{aligned}\tag{3-29}$$

From (3-29), the equilibrium equation for $\Delta\underline{x}_2^{*}$ is found, noting that:

$$\begin{aligned}\Delta\underline{x}_2^{*} &= \Delta\underline{x}_2^{*}(0) + \int\Delta\dot{\underline{x}}_2^{*}(\tau)\,d\tau\\ &= \int\Delta\dot{\underline{x}}_2^{*}(\tau)\,d\tau\end{aligned}\tag{3-30}$$

Thus

$$\Delta\underline{x}_2^{*} = K_y\int\Delta\underline{y}^{*}\,d\tau\tag{3-31}$$

A relationship for $\Delta\underline{x}_1^{*}$ is found by differentiating equation (3-18), such that:

$$\begin{aligned}\Delta\dot{\underline{x}}_1^{*} &= S_{12}\overset{0}{\cancel{\Delta\dot{\underline{y}}^{*}}} - (S_{11}F_{12} + S_{12}H_{x_2})\Delta\dot{\underline{x}}_2^{*} + S_{11}\Delta\ddot{\underline{x}}_1^{*}\\ &= -(S_{11}F_{12} + S_{12}H_{x_2})\Delta\dot{\underline{x}}_2^{*} + S_{11}\Delta\ddot{\underline{x}}_1^{*}\end{aligned}\tag{3-32}$$

Differentiating again:

$$\begin{aligned}\Delta\ddot{\underline{x}}_1^{*} &= -(S_{11}F_{12} + S_{12}H_{x_2})\overset{0}{\cancel{\Delta\ddot{\underline{x}}_2^{*}}} + S_{11}\overset{0}{\Delta\dddot{\underline{x}}_1^{*}}\\ &= 0\end{aligned}\tag{3-33}$$

Thus, from (3-33), equation (3-32) becomes:

$$\begin{aligned}\Delta\underline{x}_1^{*} &= -(S_{11}F_{12} + S_{12}H_{x_2})\Delta\dot{\underline{x}}_2^{*} + S_{11}\overset{0}{\Delta\ddot{\underline{x}}_1^{*}}\\ &= -(S_{11}F_{12} + S_{12}H_{x_2})\Delta\dot{\underline{x}}_2^{*}\\ &= -(S_{11}F_{12} + S_{12}H_{x_2})K_y\Delta\underline{y}^{*}\end{aligned}\tag{3-34}$$

By substituting (3-34) into equation (3-18), a relationship for the nonsingular variables, $\Delta\underline{x}_1^*$, in terms of $\Delta\underline{y}^*$ and $\Delta\underline{x}_2^*$ results:

$$\begin{aligned}
\Delta\underline{x}_1^* &= S_{12}\Delta\underline{y}^* - (S_{11}F_{12} + S_{12}H_{x_2})\Delta\underline{x}_2^* - S_{11}\Delta\dot{\underline{x}}_1^* \\
&= S_{12}\Delta\underline{y}^* - (S_{11}F_{12} + S_{12}H_{x_2})\Delta\underline{x}_2^* - \\
&\qquad\qquad S_{11}(S_{11}F_{12} + S_{12}h_{x_2})K_y\Delta\underline{y}^* \\
&= (S_{12} - S_{11}(S_{11}F_{12} + S_{12}H_{x_2})K_y)\Delta\underline{y}^* - \\
&\qquad\qquad (S_{11}F_{12} + S_{12}H_{x_2})\Delta\underline{x}_2^* \qquad (3\text{-}35)
\end{aligned}$$

Finally, a relationship for the equilibrium controls is found by substituting (3-34) into (3-19) as follows:

$$\begin{aligned}
\Delta\underline{u}^* &= S_{22}\Delta\underline{y}^* - (S_{21}F_{12} + S_{22}H_{x_2})\Delta\underline{x}_2^* + S_{21}\Delta\dot{\underline{x}}_1^* \\
&= S_{22}\Delta\underline{y}^* - (S_{21}F_{12} + S_{22}h_{x_2})\Delta\underline{x}_2^* - S_{21}(S_{11}F_{12} + S_{12}h_{x_2})K_y\Delta\underline{y}^* \\
&= (S_{22} - S_{21}(S_{11}F_{12} + S_{12}h_{x_2})K_y)\Delta\underline{y}^* - (S_{21}F_{12} + S_{22}H_{x_2})\Delta\underline{x}_2^* \\
&\qquad\qquad (3\text{-}36)
\end{aligned}$$

Since this CAS is designed as a sampled-data controller, the corresponding sampled-data equilibrium equations are:

$$\Delta\underline{x}_{2_k}^* = \Delta\underline{x}_{2_{k-1}}^* + K_y\Delta t\Delta\underline{y}_k^* \qquad (3\text{-}37)$$

$$\Delta\underline{x}_{1_k}^* = (S_{12} - S_{11}(S_{11}F_{12} + S_{12}h_{x_2})K_y)\Delta\underline{y}_k^* - (S_{11}F_{12} + S_{12}h_{x_2})\Delta\underline{x}_{2_k}^* \qquad (3\text{-}38)$$

$$\Delta\underline{u}_k^* = (S_{22} - S_{21}(S_{11}F_{12} + S_{12}h_{x_2})K_y)\Delta\underline{y}_k^* - (S_{11}F_{12} + S_{12}H_{x_2})\Delta\underline{x}_{2_k}^* \qquad (3\text{-}39)$$

Finally, equations (3-37) through (3-39) is simplified further by defining the following quantities:

$$SXY = S_{12} - S_{11}(S_{11}F_{12} + S_{12}H_{x_2})K_y \tag{3-40}$$

$$SXI = -(S_{11}F_{12} + S_{12}H_{x_2}) \tag{3-41}$$

$$SUY = S_{22} - S_{21}(S_{11}F_{12} + S_{12}H_{x_2})K_y \tag{3-42}$$

$$SUI = -(S_{21}F_{12} + S_{22}H_{x_2}) \tag{3-43}$$

Thus:

$$\Delta\underline{x}^*_{2_k} = \Delta\underline{x}^*_{2_{k-1}} + K_y \,\Delta t\, \Delta\underline{y}^*_k \tag{3-44}$$

$$\Delta\underline{x}^*_{1_k} = SXY\,\Delta\underline{y}^*_k + SXI\Delta\underline{x}^*_{2_k} \tag{3-45}$$

$$\Delta\underline{u}^*_k = SUY\,\Delta\underline{y}^*_k + SUI\Delta\underline{x}^*_{2_k} \tag{3-46}$$

3.2 CALCULATION OF OPTIMAL GAINS

Using sampled-data, linear-quadratic control theory, the optimal gains, C, are,

$$C = (\hat{R} + \Gamma^T P_{ss})^{-1}(\Gamma^T P_{ss}\Phi + \hat{M}^T) \tag{3-47}$$

where Φ and Γ are the sampled-data system equations, $\hat{R}$ and $\hat{M}$ are sampled-data weighting matrices and P_{ss} is the solution to the discrete Riccati equation. To solve for the optimal gains, then, the sampled-data system equations and weighting matrices and the solution to the discrete Riccati equation had to be found.

3.2.1 Sampled-Data System Equations

The continuous-time, system equations are of the following form:

$$\Delta \dot{\underline{x}} = F \, \Delta \underline{x} + G \, \Delta \underline{u} \qquad (3\text{-}48)$$

By neglecting the control effects such that $\Delta \underline{u} = \underline{0}$ (this can be done by superposition), the equation becomes:

$$\Delta \dot{\underline{x}} = F \, \Delta \underline{x} \qquad (3\text{-}49)$$

Solving for $\Delta \underline{x}$ (using the Laplace transform method), the equation reduced to:

$$(sI-F) \, \Delta \underline{x} = \Delta \underline{x}(0)$$

$$\Delta \underline{x} = (sI-F)^{-1} \, \Delta \underline{x}(0) \qquad (3\text{-}50)$$

or, in the time domain:

$$\Delta \underline{x} = \epsilon^{Ft} \, \Delta \underline{x}(0) \qquad (3\text{-}51)$$

The equivalent recursive equation for propogating the state from one instant to the next is,

$$\Delta \underline{x}(t_1) = \epsilon^{F(t_1-t_0)} \, \Delta \underline{x}(t_0) \qquad (3\text{-}52)$$

The state transition matrix, Φ, can be defined as:

$$\Phi = \epsilon^{F(t_1-t_0)} \qquad (3\text{-}53)$$

or it can be defined over an interval, Δt, such that:

$$\Delta \underline{x}(t+\Delta t) = \Phi(\Delta t) \, \Delta \underline{x}(t) \qquad (3\text{-}54)$$

The calculation of the state transition matrix involves the use of the series representation for ϵ:

$$\epsilon^{a\Delta t} = 1 + a\Delta t + (a\Delta t)^2/2 + (a\Delta t)^3/3 + \ldots \qquad (3\text{-}55)$$

In matrix notation for F:

$$\Phi(\Delta t) = 1 + F\Delta t + 1/2\ (F\Delta t)^2 + 1/3\ (F\Delta t)^3 + \ldots \qquad (3\text{-}56)$$

In the case where $\Delta\underline{u} \neq \underline{0}$:

$$\Delta\underline{x}(t) = \Phi(\Delta t)\Delta\underline{x}(t_0) + \int \Phi(\Delta t,\tau)G(\tau)\Delta\underline{u}(\tau)\ d\tau \qquad (3\text{-}57)$$

The control effects matrix, Γ, is defined as:

$$\begin{aligned} \Gamma &= \int \Phi(\Delta t,\tau)G(\tau)\ d\tau \\ &= \int \epsilon^{F(\Delta t-\tau)}\ d\tau\ G \\ &= \Phi(\Delta t)\int \epsilon^{-F\tau}\ d\tau\ G \qquad (3\text{-}58) \end{aligned}$$

Solving $\int \epsilon^{-F\tau}\ d\tau$:

$$\begin{aligned} \int \epsilon^{-F\tau}\ d\tau &= \int (1 - F\Delta t + 1/2\ (F\Delta t)^2 - 1/3\ (F\Delta t)^3 + \ldots)\ d\tau \\ &= I\Delta t - F\Delta t^2 + 1/2\ F^2\Delta t^3 - 1/3\ F^3\Delta t^4 + \ldots \\ &= \Delta t(I - F\Delta t + 1/2\ (F\Delta t)^2 - 1/3\ (F\Delta t)^3 + \ldots) \qquad (3\text{-}59) \end{aligned}$$

Therefore, the control effects matrix is found as follows:

$$\Gamma = \Phi(\Delta t)\Delta t(I - F\Delta t + 1/2\ (F\Delta t)^2 - 1/3\ (F\Delta t)^3 + \ldots)\ G \qquad (3\text{-}60)$$

3.2.2 Sampled-Data State- and Control-Weighting Matrices

The sampled-data cost function, J, defined in (3-1) as:

$$J = 1/2 \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} (\Delta \underline{x}^T Q_c \Delta \underline{x} + \Delta \underline{u}^T R_c \Delta \underline{u})\ dt$$

may also be defined as:

$$J = 1/2 \sum_{k=0}^{\infty} (\Delta \underline{x}_k^T \hat{Q} \Delta \underline{x}_k + \Delta \underline{x}_k^T \hat{M} \Delta \underline{u}_k + \Delta \underline{u}_k^T \hat{R} \Delta \underline{u}_k) \qquad (3\text{-}61)$$

where $\hat{Q}$, $\hat{M}$, and $\hat{R}$ are sampled-data, state- and control-weighting matrices and are defined in terms of the continuous-time weighting matrices as follows:

$$\hat{Q} = \int_0^{\Delta t} \Phi^T(\tau) Q_c \Phi(\tau)\ d\tau \qquad (3\text{-}62)$$

$$\hat{M} = \int_0^{\Delta t} \Phi^T(\tau) Q_c \Gamma(\tau)\ d\tau \qquad (3\text{-}63)$$

$$\hat{R} = \int_0^{\Delta t} (R_c + \Gamma^T(\tau) Q_c \Gamma(\tau))\ d\tau \qquad (3\text{-}64)$$

The integrals are solved using Simpson's rule, which is:

$$\int_a^b f(t)\ dt = (^{(b-a)}/(3n))(f(t_0) + 4f(t_1) + 2f(t_2) + \ . \ . \ .$$
$$2f(tn-2) + 4f(tn-1) + f(tn)) \qquad (3\text{-}65)$$

In this case, (b-a) is defined as the sampling time, Δt, and n is the number of subintervals in the sample (10).

The calculations are simplified noting that:

$$\Phi(\Delta t)\ \Phi(\Delta t) = \Phi(2\Delta t) \qquad (3\text{-}66)$$

Thus, only one state transition matrix (for $t = \Delta t/10$) needed to be calculated instead of calculating one at every nt (Ref. 6).

3.2.3 Solution to the Discrete Riccati Equation

The discrete Riccati equation (Ref. 7) is as follows:

$$P_{k-1} = \Phi^T P_k \Phi + \hat{Q} - (\Gamma^T P_k \Phi + \hat{M}^T)^T (\hat{R} \; \Gamma^T P_k \Gamma)^{-1} (\Gamma^T P_k \Phi + \hat{M}^T) \tag{3-67}$$

The equation is iterated until a steady-state solution is reached, i.e.:

$$P_k = P_{k-1} = P_{ss}$$

3.2.4 Computation of the Closed-Loop System

Once the optimal gains are calculated, the equivalent closed-loop system dynamics equation, F_{cl}, is found and its stability characteristics investigated. First, the closed-loop state transition matrix is found by:

$$\Phi_{cl} = \Phi - \Gamma G \tag{3-68}$$

F_{cl} is then found using the series representation for the natural log:

$$\begin{aligned} a &= (1/\Delta t)\ln(e^{a\Delta t}) \\ &= (1/\Delta t)((e^{a\Delta t} - 1) - (1/2)(e^{a\Delta t} - 1)^2 + (1/3)(e^{a\,\Delta t} - 1)^3 - \; . \; . \; . \end{aligned} \tag{3-69}$$

Therefore,

$$F_{c1} = (1/\Delta t)\ln(\Phi_{c1})$$
$$= (1/\Delta t)((\Phi - 1) - (1/2)(\Phi - 1)^2 + (1/3)(\Phi - 1)^3 - \ . \ . \ .) \qquad (3\text{-}70)$$

3.3 CALCULATION OF CONTROL GAINS

To summarize the results up to this point, the steady-state estimates ($\Delta\underline{x}_1^*$, $\Delta\underline{x}_2^*$, $\Delta\underline{u}^*$) are developed, and those equations are:

$$\Delta\underline{x}^*_{2_k} = \Delta\underline{x}^*_{2_{k-1}} + K_y \Delta t \Delta\underline{y}^*_k \qquad (3\text{-}44)$$
$$\Delta\underline{x}^*_{1_k} = SXY\ \Delta\underline{y}^*_k + SX1\ \Delta\underline{x}^*_{2_k} \qquad (3\text{-}45)$$
$$\Delta\underline{u}^*_k = SUY\ \Delta\underline{y}^*_k + SU1\ \Delta\underline{x}^*_{2_k} \qquad (3\text{-}46)$$

where all terms are previously defined. In addition, the optimal gains, C, are calculated. Therefore, by substituting into the control law:

$$\Delta\underline{u}_k = \Delta\underline{u}^*_k - C\ (\Delta\underline{x}_k - \Delta\underline{x}^*_k) \qquad (3\text{-}3)$$

the control law becomes:

$$\Delta\underline{u}_k = \Delta\underline{u}^*_k - C\Delta\underline{x}_k - C^* \begin{bmatrix} \Delta\underline{x}^*_{1_k} \\ \Delta\underline{x}^*_{2_k} \end{bmatrix}$$
$$= \Delta\underline{u}^*_k - C\Delta\underline{x}_k - C_1\Delta\underline{x}^*_{1_k} - C_2\Delta\underline{x}^*_{2_k}$$
$$= SUY\ \Delta\underline{y}^*_k + SU1\ \Delta\underline{x}^*_{2_k} - C\ \Delta\underline{x}_k - C_1(SXY\ \Delta\underline{y}^*_k + SX1\ \Delta\underline{x}^*_{2_k}) - C_2\Delta\underline{x}^*_{2_k}$$

$$= (SLY - C_1 SXY)\Delta\underline{y}^*_k + (SL1 - C_1 SX1 - C_2)\Delta\underline{x}^*_{2_k} - C\,\Delta\underline{x}_k \qquad (3\text{-}71)$$

By defining:

$$C_f = SLY - C_1\ SXY \qquad (3\text{-}72)$$

$$C_s = SL1 - C_1\ SX1 - C_2 \qquad (3\text{-}73)$$

$$C_b = -C \qquad (3\text{-}74)$$

the control law simplified to:

$$\Delta\underline{u}_k = C_f\,\Delta\underline{y}^*_k + C_s\,\Delta\underline{x}^*_{2_k} + C_b\Delta\underline{x}_k \qquad (3\text{-}75)$$

Since

$$\Delta\underline{x}^*_{2_k} = K_y\int\Delta\underline{y}^*_k\,dt \qquad (3\text{-}44)$$

The control law is rewritten as:

$$\Delta\underline{u}_k = C_f\Delta\underline{y}^*_k + C_s K_y\int\Delta\underline{y}^*_k\,dt + C_b\Delta\underline{x}_k \qquad (3\text{-}76)$$

By defining:

$$C_i = C_s K_y \qquad (3\text{-}77)$$

the final control law is then written as:

$$\Delta\underline{u} = C_f\Delta\underline{y}^*_k + C_i\int\Delta\underline{y}^*_k\,dt + C_b\Delta\underline{x}_k \qquad (3\text{-}78)$$

where $\Delta\underline{y}^*_k$ is the command, $\Delta\underline{x}_k$ is the current value of the state, and:

$$C_f = S_{22} - S_{21}(S_{11}F_{12} + S_{12}H_{x_2})K_y -$$

$$C_1(S_{12} - S_{11}(S_{11}F_{12} + S_{12}H_{x_2})K_y) \quad (3\text{-}79)$$

$$C_i = (-(S_{21}F_{12} + S_{22}H_{x_2})+C_1(S_{11}F_{12} + S_{12}H_{x_2}))K_y \quad (3\text{-}80)$$

$$C_b = -C \quad (3\text{-}81)$$

3.4 CLOSED-LOOP RESULTS

Once the control law design method was set, the selection of the continuous-time weighting matrices, Q_c and R_c, was necessary. Since these matrices affected response, desired response characteristics had to be determined before Q_c and R_c could be chosen.

After the controller was subsequently formulated, verification of its operation through simulation also needed to be carried out. Results of the verification were presented in two ways. First, a detailed description of a nominal flight condition was presented. Second, a summary of 27 different flight condition simulations was presented including closed-loop eigenvalues and response characteristics to two different commands.

3.4.1 Selection of Q_c and R_c

Before the sampled-data weighting matrices could be calculated, the continuous-time weighting matrices, Q_c and R_c, had to be selected. Since there was no method for determining the weights which give the desired step response, the selection was based on a trial-and-error iteration. Tradeoffs in response characteristics were examined and the weights which gave the "best" result were used.

Three step response characteristics were considered in the selection--rise time, defined as the amount of time to go from 10 to 90 percent of final value; overshoot, the percentage over final value that the response reached at first peak; and settling time, the amount of time to settle to within one percent of final value. The test was run using a nominal flight condition (angle of attack = 10 deg, throttle = .15, velocity = 150 feet per second) and two commands (10 deg/sec roll rate with zero sideslip and zero roll rate with 2 deg of sideslip).

The desired response was selected for two types of commands. The first was a roll rate command (with zero sideslip) for which minimum rise time, overshoot, and settling time were wanted. The second command was a sideslip command (with zero roll rate) for which minimum overshoot and settling time and a rise time around 1 sec were desirable. The selection of Q_c and R_c were based on the results closest to these criteria.

Before starting the selection process, an initial Q_c and R_c were chosen. In particular, only variations in the Q_c elements corresponding to sideslip (Q_B) and roll angle (Q_ϕ) were found to be important in arriving at suitable responses. The other elements in the weighting matrices were set to the inverses of the maximum mean-square values of the states and controls (Ref. 5). These mean values used are:

$$r = 10 \text{ deg/sec}$$

$$p = 10 \text{ deg/sec}$$

$$dR = 10 \text{ deg}$$

$$dA = 40 \text{ deg}$$

Using these values and rounding to one significant figure, the weighting matrices are:

$$Q_c = \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & Q_\beta & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & Q_\phi \end{bmatrix} \quad R_c = \begin{bmatrix} 1.0 & 0 \\ 0 & .10 \end{bmatrix}$$

The Q_β and Q_ϕ terms then were varied to find the best choice of those weights. At first, each was set at five different values (1, 25, 50, 75, 100), such that 25 different combinations were tested. The results were listed in Table 2. The roll rate command seemed to be best when Q_ϕ was 25 but the sideslip command was too slow for $Q_\beta = 1$ and too fast for $Q_\beta = 25$. Another set of weights were tested using $Q_\phi = 25$ and $Q_\beta = (1, 5, 10, 15, 20, 25)$. These results were listed in Table 3. From these tests, $Q_\beta = 10$ appeared to be the best choice.

Therefore, the final continuous-time weighting matrices are:

$$Q_c = \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 10.0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 25.0 \end{bmatrix} \quad R_c = \begin{bmatrix} 1.0 & 0 \\ 0 & .10 \end{bmatrix}$$

Table 2

Response Charactristics Varying Sideslip and Roll Angle Weightings

SIDESLIP WEIGHTING	ROLL ANGLE WEIGHTING	EIGENVALUES	$\Delta y = \begin{bmatrix}10\\0\end{bmatrix}$			$\Delta y = \begin{bmatrix}0\\2\end{bmatrix}$		
Q_β	Q_ϕ	(Dutch Roll, Roll, Spiral)	RISE TIME (sec)	OVERSHOOT (%)	SETTLING TIME (sec)	RISE TIME (sec)	OVERSHOOT (%)	SETTLING TIME (sec)
OPEN	LOOP	-.4124 ± 2.4206j -4.4127 .0513	.388	7.05	-	.51	3.55	-
1	1	-2.2533 ± .2625j -8.6816 - .9795	.174	6.97	2.70	1.47	-	3.20
1	25	-2.1375 ± .3124j -5.5066 ± 1.6542j	.152	14.65	1.15	1.48	-	3.30
1	50	-2.1206 ± .2923j -5.5782 ± 2.7347j	.149	16.07	1.00	1.48	-	3.35
1	75	-2.1080 ± .2845j -5.6270 ± 3.1518j	.147	16.81	.95	1.48	-	3.40
1	100	-2.0906 ± .2690j -5.6705 ± 3.3946j	.146	17.23	.90	1.49	-	3.40
25	1	-3.1665 ± 2.6614j -8.4946 - .9785	.174	7.11	2.70	.58	2.35	1.60
25	25	-3.2144 ± 2.5423j -5.2477 ± 1.5524j	.153	14.84	1.15	.58	2.15	1.60
25	50	-3.1569 ± 2.4779j -5.3575 ± 2.7104j	.150	16.02	1.00	.59	2.10	1.60
25	75	-3.1183 ± 2.4479j -5.4182 ± 3.1309j	.148	16.74	.95	.59	2.05	1.60
25	100	-3.0897 ± 2.4236j -5.4626 ± 3.3691j	.147	17.14	.95	.59	2.00	1.65
50	1	-3.3670 ± 3.0928j -8.4105 - .9786	.174	7.16	2.70	.50	3.20	1.45
50	25	-3.4425 ± 3.0281j -5.1833 ± 1.4623j	.153	14.94	1.15	.50	2.95	1.45

Table 2

continued

SIDESLIP WEIGHTING	ROLL ANGLE WEIGHTING	EIGENVALUES	$\Delta y = \begin{bmatrix}10\\0\end{bmatrix}$			$\Delta y = \begin{bmatrix}0\\2\end{bmatrix}$		
Q_β	Q_ϕ	(Dutch Roll, Roll, Spiral)	RISE TIME (sec)	OVERSHOOT (%)	SETTLING TIME (sec)	RISE TIME (sec)	OVERSHOOT (%)	SETTLING TIME (sec)
50	50	-3.4308 ± 2.9406j -5.2466 ± 2.6403j	.150	16.07	1.00	.50	2.90	1.45
50	75	-3.3991 ± 2.8907j -5.2982 ± 3.0745j	.148	16.79	.95	.50	2.80	1.45
50	100	-3.3716 ± 2.8565j -5.3387 ± 3.3168j	.148	17.19	.95	.50	2.80	1.50
75	1	-3.4605 ± 3.3038j -8.3633 - .9786	.174	7.18	2.70	.48	3.55	1.40
75	25	-3.5342 ± 3.2687j -5.1648 ± 1.4176j	.153	15.00	1.10	.48	3.40	1.40
75	50	-3.5512 ± 3.1940j -5.1993 ± 2.5796j	.150	16.10	1.00	.48	3.35	1.40
75	75	-3.5361 ± 3.1370j -5.2331 ± 3.0174j	.148	16.83	.95	.49	3.30	1.40
75	100	-3.5156 ± 3.0963j -5.2650 ± 3.2633j	.148	17.23	.95	.49	3.25	1.40
100	1	-3.5158 ± 3.4307j -8.3322 - .9786	.174	7.19	2.70	.48	3.85	1.35
100	25	-3.5837 ± 3.4122j -5.1570 ± 1.3942j	.153	15.04	1.10	.48	3.70	1.35
100	50	-3.6127 ± 3.3529j -5.1793 ± 2.5373j	.150	16.11	1.00	.48	3.60	1.35
100	75	-3.6116 ± 3.2987j -5.1985 ± 2.9700j	.149	16.85	.95	.48	3.55	1.35
100	100	-3.5996 ± 3.2561j -5.2210 ± 3.2160j	.148	17.25	.95	.48	3.50	1.35

Table 3

Response Characteristics Varying Sideslip Weighting

			$\Delta y = \begin{bmatrix}10\\0\end{bmatrix}$			$\Delta y = \begin{bmatrix}0\\2\end{bmatrix}$		
Sideslip Weighting β	Roll Angle Weighting φ	EIGENVALUES (Dutch Roll, Roll-Spiral)	Rise Time (sec)	Over Shoot (%)	Settling Time (sec)	Rise Time (sec)	Over Shoot (%)	Settling Time (sec)
1	25	-2.1375 ± .3124j -5.5066 ± 1.6542j	.152	14.65	1.15	1.48	-	3.30
5	25	-2.5555 ± 1.4279j -5.4384 ± 1.6474j	.152	14.68	1.15	.937	.4	1.10
10	25	-2.8270 ± 1.8860j -5.3694 ± 1.6289j	.152	14.73	1.15	.871	1.05	1.70
15	25	-3.0012 ± 2.1701j -5.3163 ± 1.6043j	.153	14.77	1.15	.666	1.05	1.70
20	25	-3.1240 ± 2.3786j -5.2768 ± 1.5778j	.153	14.81	1.15	.596	1.85	1.65
25	25	-3.2144 ± 2.5423j -5.2477 ± 1.5524J	.153	14.84	1.15	.580	2.15	1.60

3.4.2 Closed-Loop Simulation: Nominal Flight Condition

Verification of the controller was done by simulations of command responses. The nominal flight condition was used for the simulations; thus, at the nominal flight condition, the linearized system dynamic and output equations were determined as follows:

$$\begin{bmatrix} \Delta r \\ \Delta b \\ \Delta p \\ \Delta \phi \end{bmatrix} = \begin{bmatrix} -.566 & 4.266 & -.506 & 0.0 \\ -.999 & -.224 & 0.000 & 0.214 \\ 2.753 & -9.167 & -4.374 & 0.0 \\ 0.0 & 0.0 & 1.000 & 0.0 \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta B \\ \Delta p \\ \Delta \phi \end{bmatrix} + \begin{bmatrix} -5.551 & 0.545 \\ 0.070 & -.000 \\ 1.115 & -8.017 \\ 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} \Delta dR \\ \Delta dA \end{bmatrix}$$

$$\begin{bmatrix} \Delta p_c \\ \Delta b_c \end{bmatrix} = \begin{bmatrix} 0.0 & 0.0 & 1.000 & 0.0 \\ 0.0 & 1.000 & 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} \Delta r \\ \Delta b \\ \Delta p \\ \Delta \phi \end{bmatrix} + \begin{bmatrix} 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix} \begin{bmatrix} \Delta dR \\ \Delta dA \end{bmatrix}$$

where Δp_c and Δb_c were roll rate and sideslip commands, respectively. The characteristic equation was found as follows:

$$0 = (s+.4124-2.4206j)(s+.4124+2.4206j)(s+4.4127)(s-.0513)$$

Thus, from the characteristic equation, a spiral instability with a long time constant (19.5 seconds) was noted.

The continuous-time system dynamic equations were converted to the following sampled-data system equations:

$$\begin{bmatrix} \Delta r_{k+1} \\ \Delta B_{k+1} \\ \Delta p_{k+1} \\ \Delta \phi_{k+1} \end{bmatrix} = \begin{bmatrix} 0.916 & 0.425 & -.039 & 0.005 \\ -.095 & 0.956 & 0.004 & 0.021 \\ 0.252 & -.677 & 0.640 & -.006 \\ 0.013 & -.038 & 0.061 & 1.000 \end{bmatrix} \begin{bmatrix} \Delta r_k \\ \Delta B_k \\ \Delta p_k \\ \Delta \phi_k \end{bmatrix} + \begin{bmatrix} -.535 & 0.069 \\ 0.034 & -.004 \\ 0.015 & -.640 \\ 0.002 & -.035 \end{bmatrix} \begin{bmatrix} \Delta dR_k \\ \Delta dA_k \end{bmatrix}$$

The sampled-data weighting matrices were found using the sampled-data system equations and the continuous-time weighting matrices. The sampled-data weighting matrices that were calculated were summarized below:

$$\hat{Q} = \begin{bmatrix} 0.146 & -.142 & 0.033 & 0.157 \\ -.142 & 0.949 & -.073 & -.293 \\ 0.033 & -.073 & 0.074 & 0.255 \\ 0.157 & -.293 & 0.255 & 2.461 \end{bmatrix}$$

$$\hat{M} = \begin{bmatrix} -.128 & -.084 \\ 0.050 & 0.167 \\ -.008 & -.088 \\ -.070 & -.364 \end{bmatrix} \qquad \hat{R} = \begin{bmatrix} 0.312 & 0.037 \\ 0.037 & 0.226 \end{bmatrix}$$

The optimal gains and, subsequently, the control gains were calculated. The control gains were as follows:

$$C_b = \begin{bmatrix} 0.724 & -.820 & -.016 & 0.084 \\ 0.288 & -.073 & 0.001 & 2.908 \end{bmatrix}$$

$$C_f = \begin{bmatrix} -0.167 & 1.027 \\ -1.172 & -.386 \end{bmatrix} \qquad C_i = \begin{bmatrix} -0.253 & 0.0 \\ -2.898 & 0.0 \end{bmatrix}$$

The equivalent closed-loop system characteristic equation was found from the optimal gains and the sampled-data system equations; it is:

$$C = (s+2.627-1.886j)(s+2.627+1.886j)(s+5.369-1.629j)$$
$$(s+5.369+1.629j)$$

As can be seen, the closed-loop system had no instabilities and fairly quick time constants. Also, the roll and spiral modes were no longer separate. Figure 2 shows a plot of the open- and closed-loop eigenvalues for comparison.

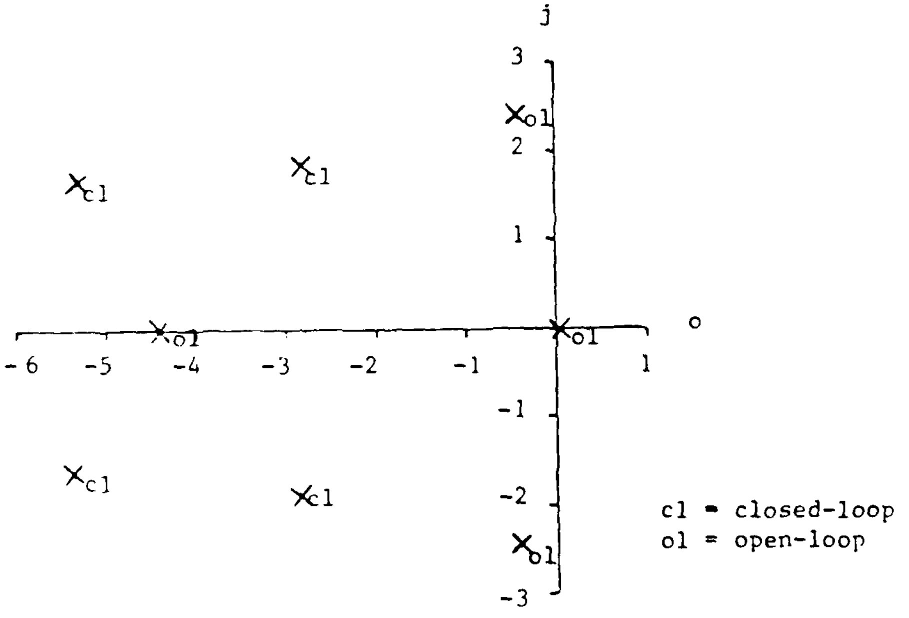


Figure 2: Eigenvalue Plot: Open-Loop vs. Closed-Loop

Finally, simulations were run to insure the proper operation of the control system. Linear simulations using state transition matrices were used for ease of comparison with later simulations.

Figure 3 shows the response of the aircraft for a roll rate command of 10 degrees per second. The rise time for the roll rate response is 0.152 seconds while the settling time was 1.15 seconds. Overshoot was 14.73 percent over final value. The sideslip experienced some steady-state error even though commanded to be zero but that error was negligible (0.000 degrees). The yaw rate response demonstrated that when command equilibrium was reached, even nonsingular variables do not necessarily reach equilibrium.

Figure 4 shows the system response for a sideslip command of 2 degrees. Rise time for the sideslip response was 0.751 seconds while settling time is 1.70 seconds. Overshoot was limited to only 1.05 percent over final value. All variables reached some steady-state value, as opposed to the roll rate command response, since the sideslip command was not singular.

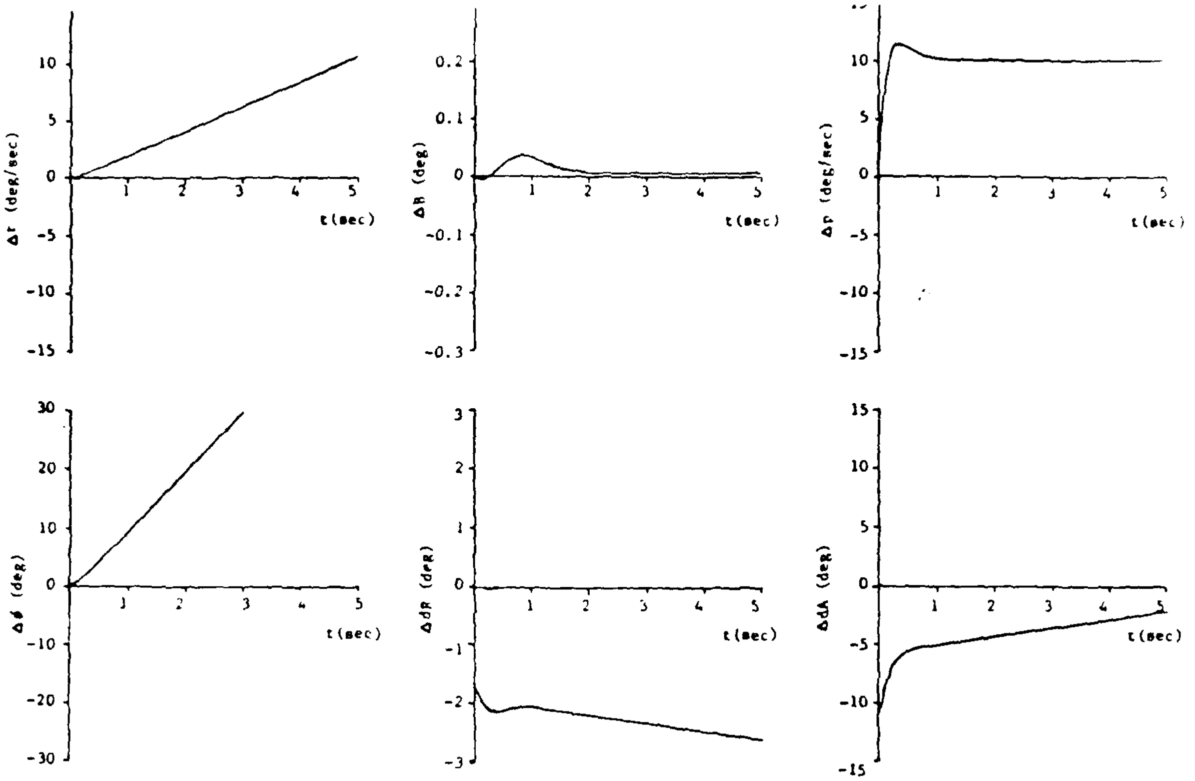


Figure 3 : Closed-Loop Simulation - Roll Rate Command

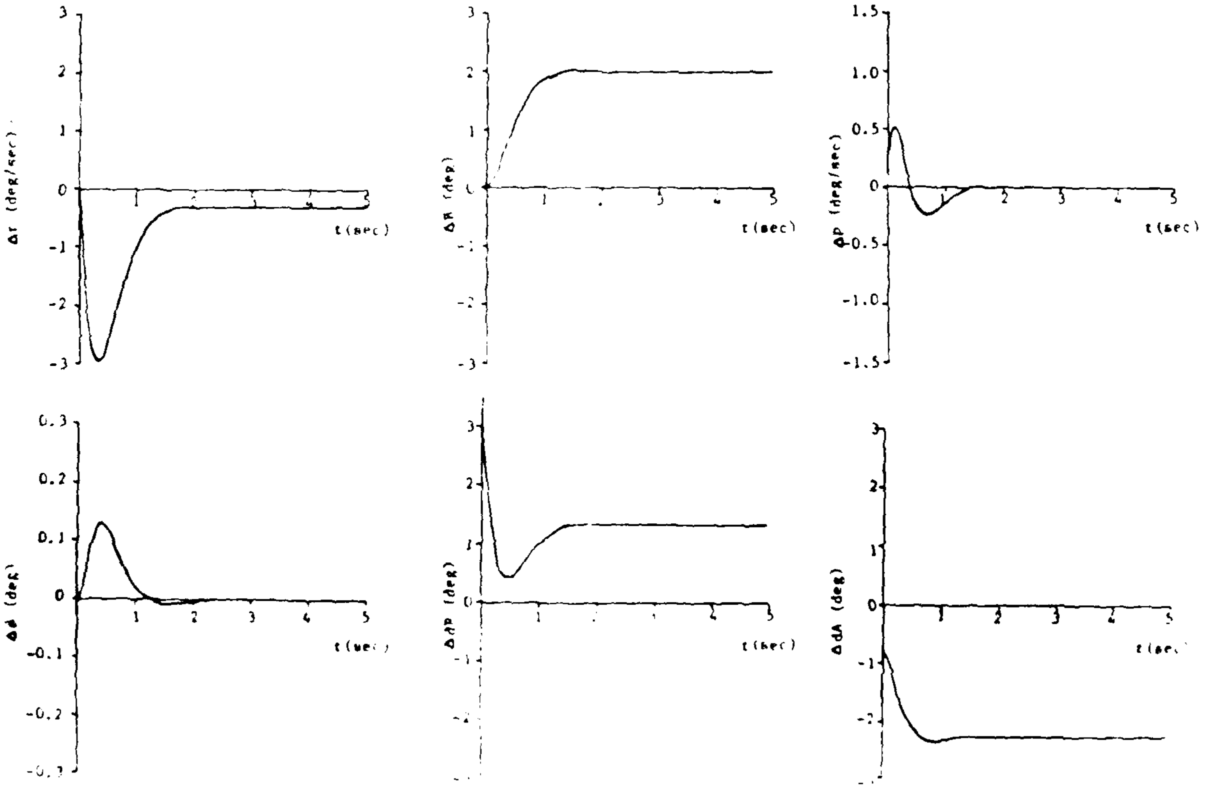


Figure 4. Closed-Loop Simulation - Sideslip Command

3.4.3 Summary of Closed-Loop Results

In addition to the nominal flight condition, 26 other cases were run using three values of each flight condition variable. The results were listed in Table 4. Listed were the closed-loop eigenvalues and the response characteristics for two command vectors.

The two commands were chosen for typical values which may be encountered in actual flight testing. In particular, a roll rate of 10 deg/sec with zero sideslip and a sideslip of 2 deg with zero roll rate were chosen.

The response charateristics of interest are: rise time, the time to go from 10 to 90 percent of final value; overshoot, the percentage over final value at the first peak; settling time, the time required to settle to within 1 percent of final value; steady-state error in roll rate, as a percentage of final value for a roll rate command and as an absolute number for a sideslip command; and steady-state error in sideslip, as an absolute number for a roll rate command and as a percentage of final value for a sideslip command.

Table 4

Closed-Loop Results

FLIGHT CONDITION			EIGENVALUES	RESPONSE $\Delta y = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$					RESPONSE $y = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$				
α (DEG)	T_c	$\bar{q}$ (lb/ft^2)	Dutch Roll Roll-Spiral	Rise Time (sec)	Overshoot (%)	Settling Time (sec)	Steady State Error P (%)	Steady State Error β (deg)	Rise Time (sec)	Overshoot (%)	Settling Time (sec)	Steady State Error P (deg/sec)	Steady State Error β (%)
-4	.03	9.731	-1.4797 ± 1.7081j -4.5205 ± 1.9247j	.161	15.78	1.20	0	.011	.823	7.50	2.75	0	.05
-4	.03	21.894	-2.2968 ± 2.2659j -5.4832 ± 1.1206j	.152	14.23	1.10	0	.006	.663	4.15	2.05	0	.05
-4	.03	38.922	-3.0265 ± 2.4610j -7.9104,- 4.5891	.148	13.66	1.05	0	.004	.597	2.05	1.65	0	0
-4	.13	9.731	-1.7252 ± 1.8320j -4.4837 ± 1.9563j	.162	15.85	1.20	0	.009	.822	5.20	2.60	0	0
-4	.13	21.894	-2.5920 ± 2.1860j -5.4084 ± 1.2512j	.153	14.36	1.10	0	.005	.679	2.40	1.95	0	0
-4	.13	38.922	-3.3053 ± 2.2722j -7.6598,- 4.6443	.148	13.77	1.05	0	.004	.662	.95	.90	0	0
-4	.23	9.731	-1.9354 ± 1.8470j -4.4479 ± 1.9877j	.162	15.91	1.20	0	.008	.823	3.75	2.45	0	0
-4	.23	21.894	-2.8180 ± 2.0989j -5.3324 ± 1.3711j	.153	14.48	1.10	0	.005	.685	1.45	1.80	0	0
-4	.23	38.922	-3.5194 ± 2.1048j -7.3650,- 4.6839	.148	13.85	1.05	0	.004	.673	.45	.80	0	0
10	.03	9.731	-1.7604 ± 1.6023j -4.4552 ± 2.0454j	.161	16.15	1.20	.04	.012	.915	3.40	2.75	0	0
10	.03	21.894	-2.5788 ± 1.8842j -5.4218 ± 1.4695j	.152	14.56	1.15	.04	.007	.760	1.50	2.00	0	0
10	.03	38.922	-3.1336 ± 1.9070j -7.5308,- 5.1357	.147	13.99	1.05	.04	.005	.687	.70	.85	0	0
10	.13	9.731	-2.0346 ± 1.7041j -4.4126 ± 2.0939j	.161	16.27	1.20	.05	.010	.844	2.55	2.45	0	0
10	.13	21.894	-2.8270 ± 1.8860j -5.3694 ± 1.6289j	.152	14.73	1.15	.05	.006	.751	1.05	1.70	0	0

Table 4

continued

FLIGHT CONDITION			EIGENVALUES	RESPONSE $\Delta y = \begin{bmatrix}10\\0\end{bmatrix}$					RESPONSE $y = \begin{bmatrix}0\\2\end{bmatrix}$				
α (DEG)	T_c	q (lb/ft^2)	Dutch Roll Roll-Spiral	Rise Time (sec)	Overshoot (%)	Settling Time (sec)	Steady State Error P (%)	Steady State Error β (deg)	Rise Time (sec)	Overshoot (%)	Settling Time (sec)	Steady State Error P (deg/sec)	Steady State Error β (%)
10	.13	38.922	-3.2743 ± 1.8810j -7.0280, - 5.5406	.148	14.06	1.05	.05	.004	.681	.55	.85	0	0
10	.23	9.731	-2.2582 ± 1.7584j -4.3712 ± 2.1450j	.162	16.38	1.20	.05	.009	.835	1.95	2.25	0	0
10	.23	21.894	-2.9833 ± 1.8696j -5.3235 ± 1.7775j	.153	14.89	1.10	.05	.005	.694	.80	.85	0	0
10	.23	38.922	-3.3465 ± 1.8689j -6.2365 ± .5758j	.148	14.14	1.10	.05	.004	.677	.50	.80	0	0
24	.03	9.731	-1.2879 ± 1.2096j -3.5074 ± 2.3667j	.177	17.94	1.35	.06	.025	1.25	3.50	3.70	.001	.1
24	.03	21.894	-1.9609 ± 1.3669j -3.9132 ± 2.6147j	.168	16.70	1.30	.06	.007	1.03	1.10	2.45	0	0
24	.03	38.922	-2.3413 ± .8606j -4.6852 ± 3.0252j	.160	15.65	1.50	.06	-.001	1.17	-	1.35	0	0
24	.13	9.731	-1.4211 ± 1.2856j -3.4911 ± 2.3133j	.178	17.97	1.35	.07	.020	1.17	3.10	3.40	0	.1
24	.13	21.894	-2.0789 ± 1.4265j -3.9289 ± 2.5560j	.170	16.67	1.40	.07	.003	.958	1.00	1.15	0	0
24	.13	38.922	-2.3614 ± 1.0300j -4.7409 ± 2.8196j	.162	15.17	1.55	.07	-.005	1.15	.05	1.30	0	0
24	.23	9.731	-1.5393 ± 1.3444j -3.4526 ± 2.2971j	.179	17.99	1.40	.08	.017	1.10	2.75	3.20	0	0
24	.23	21.894	-2.1530 ± 1.4844j -3.9561 ± 2.4773j	.172	16.66	1.45	.08	.001	.949	.95	1.15	0	0
24	.23	38.922	-2.3306 ± 1.2170j -4.7904 ± 2.5881j	.165	14.66	1.60	.08	-.008	1.07	.20	1.20	0	0

Chapter IV

GAIN SCHEDULING

Up to this point, the control system consisted of 27 different flight conditions and hence 27 different sets of gains. The problem was to find a scheme to schedule the gains such that the microprocessor would have the correct set of gains for the particular flight condition. The limitations of the microprocessor and the requirements of the CAS each had an effect on the eventual gain scheduling solution.

As mentioned above, the microprocessor limitations affected the form of the gain schedules. In particular, the memory space afforded the CAS program was limited to about 28K bytes. In addition, the microprocesser could only do a limited number of calculations during any interval of time. It was desirable to continuously update the gains to account for changes in the flight condition, but all calculations would have to be done within the sampling time, 0.1 seconds. Hence, the gain schedule had to be as small as possible without sacrificing accuracy, which the CAS required for proper operation.

Two methods could have been used for gain scheduling. The first was a table lookup method where the computer senses the flight condition and looks up the appropriate gains. In order for this scheme to work effectively, there would have had to be a large table covering the variety of flight conditions which the aircraft might encounter. Such a method was not possible, since it required large memory space not available in the microprocessor. Therefore, this method of scheduling was rejected.

The second method involved calculating the gains as functions of the flight condition. This method included investigation into the sensitivities of the gains to changes in the flight condition and selection of a suitable solution form to match those sensitivities. By using the same solution form for as many gains as possible, the coefficients could be put into a set of matrices to simplify calculations and reduce the computation time. This method, too, had its drawbacks, in that if the gains were to be updated every sampling time, a scheme had to be developed to do all calculations within the sampling time. However, the drawbacks in this method did not seem as serious as those of the other method; therefore, this method was selected.

This chapter covers the formulation of the gain equations. In particular the flight condition functions, which are used in the gain equations, are discussed, as is the selection of the solution forms for the flight condition functions. In addition, the gain coefficient matrix computation method is covered. Finally, results are included showinq a simulation for a nominal flight condition using the gain schedules.

4.1 FLIGHT CONDITION FUNCTIONS

At each flight condition, a different set of gains was required to provide satisfactory control of the aircraft. Thus, the gains were functions of the flight condition variables and could be represented as follows:

$$G = f(\alpha)\ f(T_c)\ f(\bar{q}) \qquad (4\text{-}1)$$

The gain scheduling task was to find suitable flight condition functions--$f(\alpha)$, $f(T_c)$, $f(\bar{q})$--whose solutions were accurate compared to the actual gains at any condition.

The flight condition functions were required to reflect the sensitivities of the gains to each flight condition variable. By investigating these sensitivities, it was possible to narrow down the different solution forms. This was done by plotting the gains versus one flight condition variable, holding both of the other flight condition variables constant.

It was desirable to find solution forms which could be used to schedule more than a single gain. By doing this, implementation of the schedules in the CAS was simpler and matrix manipulation could be done with less memory space than otherwise. Therefore, selection of flight condition functions was limited to those which could be used for several gains.

The data used for gain scheduling included the 27 sets of gains and flight conditions reflecting all possible combinations of the three values for each flight condition. The results are presented in Table 5. Since only three values of each flight condition variable were used, the polynomial functions,

$$f(a) = B_0 + B_1\, a^1 + B_2\, a^2 \qquad (4\text{-}2)$$

$$f(T_c) = C_0 + C_1\, T_c^1 + C_2\, T_c^2 \qquad (4\text{-}3)$$

$$f(\bar{q}) = D_0 + D_1\, \bar{q}^1 + D_2\, \bar{q}^2 \qquad (4\text{-}4)$$

scheduled the gains exactly. (Any three points can be described by a second-order function). When these flight condition functions were multiplied in (4-1) and put into matrix form, a gain coefficient matrix of 14 by 27 resulted. Though this was not necessarily too large for use in the microprocessor, it did require a lot of computation time and a lot of memory space that could have been used more effectively by the CAS. It was felt that it would be better to reduce computation time and accept some error in the gains than to risk not being able to do the required computations within the sampling interval. Nevertheless, the exact solution did give a starting point for comparison.

In addition, a correlation factor was computed for the gain schedules to determine how well the gain schedule approximated the actual gains. The factor was computed as follows:

$$\text{CORRELATION} = 1 - \left(\left(C_{actual} - C_{scheduled}\right)/C_{actual}\right)^2 \qquad (4\text{-}5)$$

It was found that a correlation below 0.8 resulted in simulations which did not reach command equilibrium, while correlations above that figure gave good results. Using the second-order solutions for all flight condition functions (as discussed above) resulted in a correlation of .9996 -- the error due to roundoff.

To facilitate computation of the gain equations, the gains were numbered as follows:

$$C_b = \begin{bmatrix} C1 & C2 & C3 & C4 \\ C5 & C6 & C7 & C8 \end{bmatrix} \quad C_f = \begin{bmatrix} C9 & C10 \\ C11 & C12 \end{bmatrix} \quad C_i = \begin{bmatrix} C13 & 0 \\ C14 & 0 \end{bmatrix}$$

Table F

Flight Condition and Gain Matrices

```
FC EQUALS:

-4.000 -4.000 -4.000 -4.000 -4.000 -4.000 -4.000 -4.000 -4.000 10.000 10.000 10.000 10.000 10.000 10.0
 0.030  0.030  0.030  0.130  0.130  0.130  0.230  0.230  0.230  0.030  0.030  0.030  0.130  0.130  0.1
 9.731 21.894 38.922  9.731 21.894 38.922  9.731 21.894 38.922  9.731 21.894 38.922  9.731 21.894 38.92

                      10.000 10.000 10.000 24.000 24.000 24.000 24.000 24.000 24.000 24.000 24.000 24.0
                       0.230  0.230  0.230  0.030  0.030  0.030  0.130  0.130  0.130  0.230  0.230  0.2
                       9.731 21.894 38.922  9.731 21.894 38.922  9.731 21.894 38.922  9.731 21.894 38.9


C EQUALS:

 1.371  0.880  0.587  1.275  0.781  0.498  1.183  0.695  0.429  1.435  0.883  0.561  1.250  0.724  0.439
-1.245 -0.662 -0.131 -1.265 -0.616 -0.095 -1.245 -0.557 -0.064 -1.685 -1.070 -0.536 -1.527 -0.826 -0.327
-0.071 -0.055 -0.050 -0.080 -0.057 -0.049 -0.086 -0.057 -0.048  0.026 -0.012 -0.024  0.015 -0.016 -0.023
-0.526 -0.450 -0.434 -0.544 -0.434 -0.399 -0.551 -0.414 -0.367  0.222  0.131  0.073  0.154  0.084  0.048
 0.424  0.324  0.238  0.640  0.442  0.307  0.821  0.534  0.358  0.342  0.270  0.198  0.415  0.288  0.205
-1.572 -1.459 -1.277 -1.884 -1.636 -1.372 -2.177 -1.793 -1.458 -0.601 -0.776 -0.785 -0.565 -0.673 -0.670
 1.328  0.593  0.324  1.320  0.586  0.322  1.311  0.580  0.318  1.316  0.597  0.339  1.324  0.601  0.341
 5.252  2.962  1.971  5.097  2.850  1.897  4.946  2.746  1.824  5.136  2.946  2.029  5.088  2.908  1.999
-0.017  0.065  0.077  0.003  0.061  0.068  0.017  0.058  0.061 -0.470 -0.223 -0.137 -0.357 -0.167 -0.105
 2.930  2.342  1.795  2.668  2.006  1.463  2.461  1.753  1.232  2.541  1.916  1.364  2.347  1.627  1.105
-2.320 -1.236 -0.802 -2.293 -1.210 -0.783 -2.263 -1.186 -0.764 -2.212 -1.188 -0.780 -2.192 -1.172 -0.768
 0.613  0.506  0.323  0.990  0.745  0.474  1.304  0.917  0.569 -0.552 -0.371 -0.363 -0.496 -0.386 -0.380
 0.023  0.234  0.324  0.085  0.245  0.307  0.130  0.247  0.288 -0.728 -0.340 -0.175 -0.589 -0.253 -0.127
-5.421 -3.046 -2.017 -5.337 -2.960 -1.955 -5.244 -2.876 -1.891 -5.100 -2.938 -2.024 -5.061 -2.898 -1.991
                       1.107  0.609  0.358  1.917  1.260  0.851  1.787  1.140  0.735  1.678  1.041  0.0
                      -1.366 -0.633 -0.165 -2.345 -1.870 -1.194 -2.244 -1.702 -1.029 -2.145 -1.552 -0.8
                       0.007 -0.018 -0.021  0.207  0.014 -0.086  0.188  0.005 -0.081  0.174  0.003 -0.0
                       0.111  0.058  0.036 -0.614 -0.972 -1.177 -0.587 -0.914 -1.054 -0.562 -0.846 -0.9
                       0.464  0.299  0.206  0.281  0.208  0.175  0.370  0.247  0.205  0.443  0.277  0.2
                      -0.495 -0.563 -0.567 -1.820 -1.930 -2.083 -1.355 -1.458 -1.625 -0.878 -0.981 -1.1
                       1.332  0.605  0.344  1.785  0.885  0.589  1.613  0.899  0.594  1.838  0.910  0.5
                       5.038  2.870  1.966  4.510  2.550  1.960  4.425  2.480  1.870  4.344  2.404  1.7
                      -0.288 -0.133 -0.083 -0.190  0.110  0.210 -0.171  0.101  0.186 -0.157  0.090  0.1
                       2.165  1.406  0.931  2.907  2.429  1.740  2.797  2.249  1.554  2.695  2.090  1.4
                      -2.171 -1.157 -0.756 -1.918 -1.005 -0.688 -1.899 -0.980 -0.661 -1.877 -0.954 -0.0
                      -0.476 -0.410 -0.411 -1.240 -1.127 -0.972 -1.196 -1.092 -0.922 -1.167 -1.067 -0.8
                      -0.492 -0.200 -0.100 -0.139  0.642  1.006 -0.096  0.621  0.908 -0.068  0.583  0.8
                      -5.012 -2.856 -1.955 -4.311 -2.466 -1.916 -4.239 -2.397 -1.827 -4.161 -2.319 -1.7
```

4.2 SELECTION OF SOLUTION FORMS

The plots of the gain sensitivities are shown in Figures 5 through 7. The plots were made by holding two flight condition variables constant and plotting the variation of the gains versus variations in the third flight condition variable. Figure 5 shows the angle of attack sensitivities of the gains. In general, the gains followed no particular pattern; hence, it seemed simplest to use the second-order solution discussed previously.

Figure 6 shows the gain variations with respect to throttle setting. All gains appeared to be linear in throttle (due to the linear assumption used in the model formulation). Thus, an appropriate solution was a first-order equation:

$$f(T_c) = C_0 + C_1 T_c \qquad (4-6)$$

In addition, a lot of the gains sloped towards zero as throttle setting increased (implying proportionality to the inverse of T_c). Since several gains showed this characteristic, another possible solution form was:

$$f(T_c) = e^{-C_0 T_c} \qquad (4-7)$$

Indeed, this turned out to be the case for half the gains. (In those gains where this form was appropriate, the constant term, C_0, had to be the same for every gain. It was found that C_0 = 0.001 was satisfactory).

Figure 7 shows the gain sensitivities to changes in dynamic pressure. Almost all the gains behaved similarly. In particular, the gains curve toward zero as dynamic pressure increased. Such a form suggested a solution of:

$$f(\bar{q}) = {}^{D_0}/\bar{q} \qquad (4-8)$$

or

$$f(\bar{q}) = e^{-D_0\bar{q}} \qquad (4-9)$$

Both (4-8) and (4-9) required constants which had to be the same for at least several gains or the solution form was not desirable. However, constants could not be found in either case which would schedule any more than a few gains. Hence, the solution form used for the dynamic pressure equation was the second-order solution previously discussed.

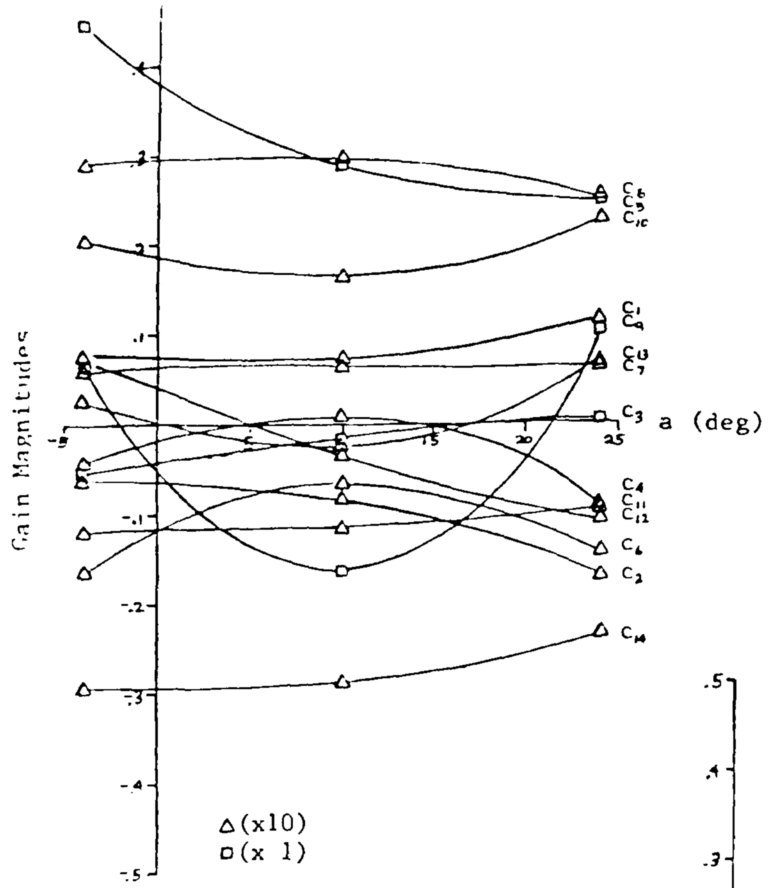


Figure 5: Gain Sensitivities to Changes
in Angle-of-Attack

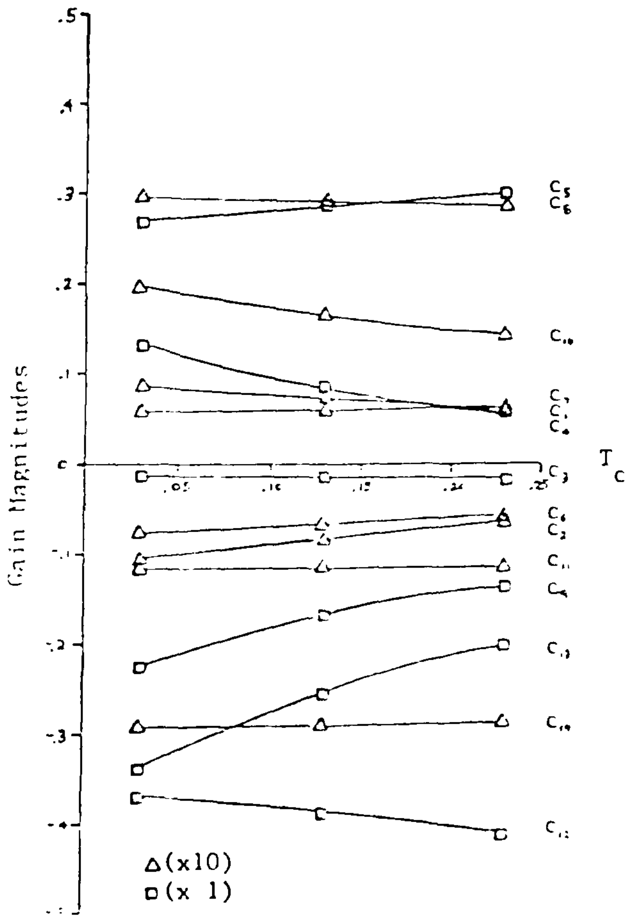


Figure 6: Gain Sensitivities to Changes
in Throttle Setting

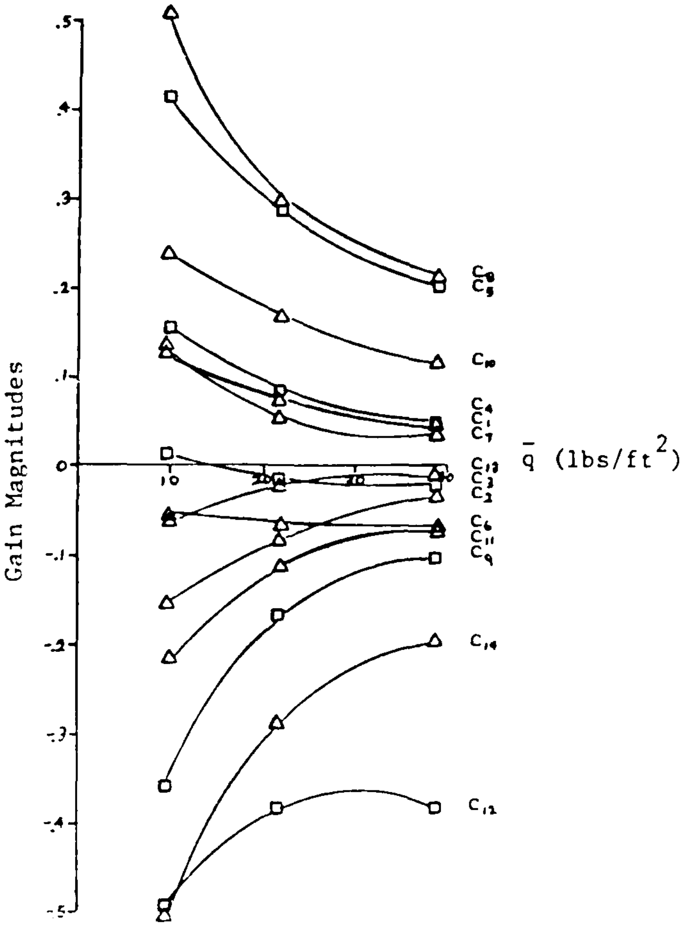


Figure 7: Gain Sensitivities to Changes in Dynamic Pressure

Thus the two solution forms for the throttle function resulted in two gain equations -- each solution form was used for half the gains. The forms are as follows:

1. For gains 1, 5, 7, 8, 9, 11, and 12:

$$C = (B_0 + B_1 a + B_2 a^2) e^{-C_0 T_c} (D_0 + D_1 \bar{q} + D_2 \bar{q}^2) \qquad (4\text{-}10)$$

2. For gains 2, 3, 4, 6, 10, 13, and 14:

$$C = (B_0 + B_1 a + B_2 a^2)(C_0 + C_1 T_c)(D_0 + D_1 \bar{q} + D_2 \bar{q}^2) \qquad (4\text{-}11)$$

When the equations were multiplied out, they took the form:

$$\begin{aligned}C = {} & A_0 e^{C_0 T_c} + A_1 e^{C_0 T_c}\,\bar{q}_T + A_2\, e^{C_0 T_c}\,\bar{q}^2 + \\ & A_3\, e^{C_0 T_c}\, a + A_4\, e^{C_0 T_c}\,\bar{q}\, a + A_5\, e^{C_0 T_c}\,\bar{q}^2\, a + \\ & A_6\, e^{C_0 T_c}\, a^2 + A_7\, e^{C_0 T_c}\,\bar{q}\, a^2 + A_8\, e^{C_0 T_c}\,\bar{q}^2\, a^2 \end{aligned} \qquad (4\text{-}12)$$

and

$$\begin{aligned}C = {} & A_0 + A_1\, T_c + A_2\,\bar{q} + \\ & A_3\, T_c\,\bar{q} + A_4\,\bar{q}^2 + A_5\, T_c\,\bar{q} + \\ & A_6\, a + A_7\, T_c\, a + A_8\,\bar{q}\, a + \\ & A_9\, T_{c_2}\bar{q}\, a + A_{10}\,\bar{q}^2\, a + A_{11}\, T_c\,\bar{q}\, a + \\ & A_{12}\, a + A_{13}\, T_c\, a^2 + A_{14}\,\bar{q}\, a + \\ & A_{15}\, T_c\,\bar{q}\, a^2 + A_{16}\,\bar{q}^2\, a^2 + A_{17}\, T_c\,\bar{q}^2\, a^2 \end{aligned} \qquad (4\text{-}13)$$

In this form, the equations could be written as a matrix equation:

$$\underline{C} = [A]\ \underline{fc} \qquad (4\text{-}14)$$

where $\underline{C}$ is the gain matrix (14 x 7 in either case), A contains the coefficients of the gain equations (14 x 9 for equation (4-12) and 14 x 18 for equation (4-13), and $\underline{fc}$ is the flight condition vector consisting of all the combinations of the flight condition variables (9 x 1 for equation (4-12) and 18 x 1 for equation (4-13).

4.3 COMPUTATION OF THE COEFFICIENT MATRICES

Once the flight condition functions were determined, the coefficients of the gain equations, [A], could be found. To simplify the calculations, all the gains with the same form of gain equation were assembled into a matrix equation; thus only two matrix manipulations needed to be accomplished -- one for each gain equation. The matrices were set up as follows:

$$[\underline{C}_{fc_1}\ \underline{C}_{fc_2}\ \cdots\ \underline{C}_{fc_{27}}] = [A][\underline{fc}_1\ \underline{fc}_2\ \cdots\ \underline{fc}_{27}] \qquad (4\text{-}15)$$

or more simply

$$[C] = [A][FC] \qquad (4\text{-}16)$$

where the column vectors of C are the gains at one particular flight condition, the column vectors of FC are the flight condition combinations for one particular flight condition, and A contains the coefficients of the equations. Since FC was not necessarily square, the solution of A involved a "pseudo-inverse" of FC (Ref. 8). The pseudo-inverse was defined as follows:

$$FC^{\#} = FC^{T}\ (FC\ FC^{T})^{-1} \qquad (4\text{-}17)$$

such that:

$$[A] = [C][fc]^{\#}$$

The final solution of the two coefficient matrices, A_1 and A_2, is presented in Tables 6 and 7. The matrix A_1

Table 6

Coefficient Gain Matrices

Matrix A_1

```
0.1790E+01 -.6704E-01 0.8409E-03 -.7062E-02 -.2718E-03 0.5783E-05 0.1574E-02 -.2029E-04 0.5743E-07
0.7297E+00 -.2105E-01 0.2326E-03 -.2591E-01 0.8608E-03 -.1064E-04 0.7078E-03 -.3056E-04 0.4830E-06
0.2155E+01 -.1052E+00 0.1480E-02 -.1192E-01 0.5802E-03 -.8376E-05 0.1896E-02 -.7971E-04 0.1183E-05
0.7689E+01 -.3227E+00 0.4372E-02 0.5739E-02 0.4320E-03 -.7653E-05 -.1968E-02 0.3013E-04 0.1268E-06
0.3254E+01 -.8524E-01 0.9080E-03 -.2486E-01 -.1404E-02 0.2740E-04 0.1056E-02 0.1207E-03 -.2462E-05
-.3595E+01 0.1556E+00 -.2146E-02 0.9345E-02 -.6244E-03 0.9687E-05 0.5536E-03 -.5157E-05 -.9062E-07
0.5348E+00 -.6276E-02 -.7038E-04 -.1531E+00 0.4123E-02 -.4765E-03 0.3235E-02 -.1460E-03 0.2176E-05
```

Matrix A_2

```
-.1986E+01 -.2263E+00 0.6908E-01 0.1142E+00 -.6239E-03 -.2183E-02 -.3905E-01 0.1234E+00 0.8762E-03
-.5978E-01 -.1526E+00 0.1118E-02 0.9144E-02 -.1153E-04 -.1301E-03 0.8944E-02 -.2450E-03 -.3950E-03
-.1687E+00 -.6572E+00 0.5632E-02 0.3318E-01 -.9288E-04 -.4252E-03 0.9175E-01 -.3717E-01 -.1522E-02
-.9162E+00 -.3385E+01 -.1312E-01 0.1550E+00 0.3222E-03 -.2074E-02 0.1234E+00 0.3041E+00 -.3296E-02
-.4223E+00 0.9430E+00 0.2269E-01 -.4901E-01 -.3330E-03 0.7071E-03 -.6199E-01 0.1051E+00 0.1961E-02
-.6580E+00 0.1412E+01 0.3657E-01 -.6358E-01 -.5010E-03 0.8178E-03 -.8771E-01 0.6820E-01 0.1415E-02
-.6224E+01 0.6892E+00 0.3341E+00 0.1572E-02 -.4504E-02 -.1901E-03 0.3630E-01 -.5388E-01 -.2292E-02
```

```
0.3238E-02 -.1553E-04 -.7741E-04 0.3322E-03 -.4530E-02 -.9593E-04 -.1424E-03 0.1853E-05 0.3766E-
-.7989E-04 0.6372E-05 0.1957E-05 0.4981E-03 -.1815E-03 -.3247E-04 0.5954E-05 0.3536E-06 0.4980E-
-.1522E-02 0.2116E-04 0.2149E-04 -.3840E-02 0.2660E-02 -.3302E-04 0.5423E-04 0.3215E-06 0.2887E-
-.6569E-02 0.4196E-04 0.4993E-04 -.6807E-02 0.1183E-02 0.1429E-03 0.3287E-04 -.2300E-05 0.9122E-
-.3469E-02 -.2631E-04 0.4469E-04 0.2308E-02 -.5170E-02 -.4179E-04 0.1639E-03 0.5309E-06 -.2211E-
0.8433E-04 -.2001E-04 -.2753E-05 0.2882E-02 -.3604E-02 0.7385E-04 -.1119E-04 -.8737E-06 -.5247E-
0.1104E-02 0.3346E-04 -.8684E-05 0.1139E-02 0.2541E-02 0.2660E-05 -.7000E-04 -.7202E-06 0.1551E-
```

(where $f(T_c) = e^{C_0 T_c}$) was 7 by 9 while A_2 (where $f(T_c) = C_0 + C_1 T_c$) was 7 by 18. Taken together, the matrices reflected a reduction of one half over the exact solution. The average correlation was .9089 with the lowest being .8200 on gain 12.

4.4 GAIN SCHEDULING SIMULATION

In order to verify the accuracy of the gain schedules, a simulation was run at the nominal flight condition using the gain schedules, and the results were compared to those results obtained using the actual gains. Some slight differences were noted between the actual and scheduled gains and, consequently, the closed-loop eigenvalues and responses were slightly different.

The actual gains and the schedule gains at the nominal flight condition are listed below for comparison.

Actual Gains	Scheduled Gains
$C_b = \begin{matrix} 0.724 & -.826 & -.016 & 0.084 \\ 0.288 & -.673 & 0.601 & 2.908 \end{matrix}$	$C_b = \begin{matrix} 0.739 & -.846 & -.015 & 0.089 \\ 0.286 & -.672 & 0.601 & 2.910 \end{matrix}$
$C_f = \begin{matrix} -0.167 & 1.627 \\ -1.172 & -.386 \end{matrix}$	$C_f = \begin{matrix} -0.174 & 1.650 \\ -1.174 & -.389 \end{matrix}$
$C_i = \begin{matrix} -0.253 & 0.0 \\ -2.898 & 0.0 \end{matrix}$	$C_i = \begin{matrix} -0.263 & 0.0 \\ -2.900 & 0.0 \end{matrix}$

In cross checking the individual gains, small differences were noted between the actual and scheduled gains. This, in turn, led to the expectation that there would be differences, though hopefully small, in the eigenvalues and, hence, responses. The characteristic equations for the actual and scheduled closed-loop systems were as follows:

Actual Gains

$$C = (s+2.827-1.886j)(s+2.827+1.886j)(s+5.369-1.629j)(s+5.369+1.629j)$$

Scheduled Gains

$$C = (s+2.888-1.848j)(s+2.888+1.848j)(s+5.370-1.656j)(s+5.370+1.656j)$$

As expected, the eigenvalues did show only slight variations; thus, the scheduled gains give a good representation of the actual ones.

Since the gains and the eigenvalues of the actual and scheduled gain systems were only slightly different, it was reasonable to expect only slight variations when comparing the responses of the two. Figure 8 shows the response of the system to a roll rate command of 10 degrees per second. The rise time for the roll rate response was 0.152 seconds compared to 0.152 seconds for the actual gains. Settling time was 1.15 seconds compared to 1.15 seconds and overshoot was 14.91 percent over final value compared to 14.73. So the roll rate responses were almost identical. In addition, the gain schedule sideslip response did not reach steady-state as it did in the actual gain case. The rate of change is very small, however, so that the disequilibrium is not a significant factor.

Figure 9 shows the system response to a sideslip command of 2 degrees. Rise time for the sideslip response was 0.766 seconds compared to 0.751 seconds in the actual gain case. Settling time was 0.90 seconds compared to 1.70 seconds and overshoot was 0.95 percent over final value compared to 1.05. The settling time difference was attributable to the smaller overshoot and the way settling time is defined (time to within 1 percent of final value).

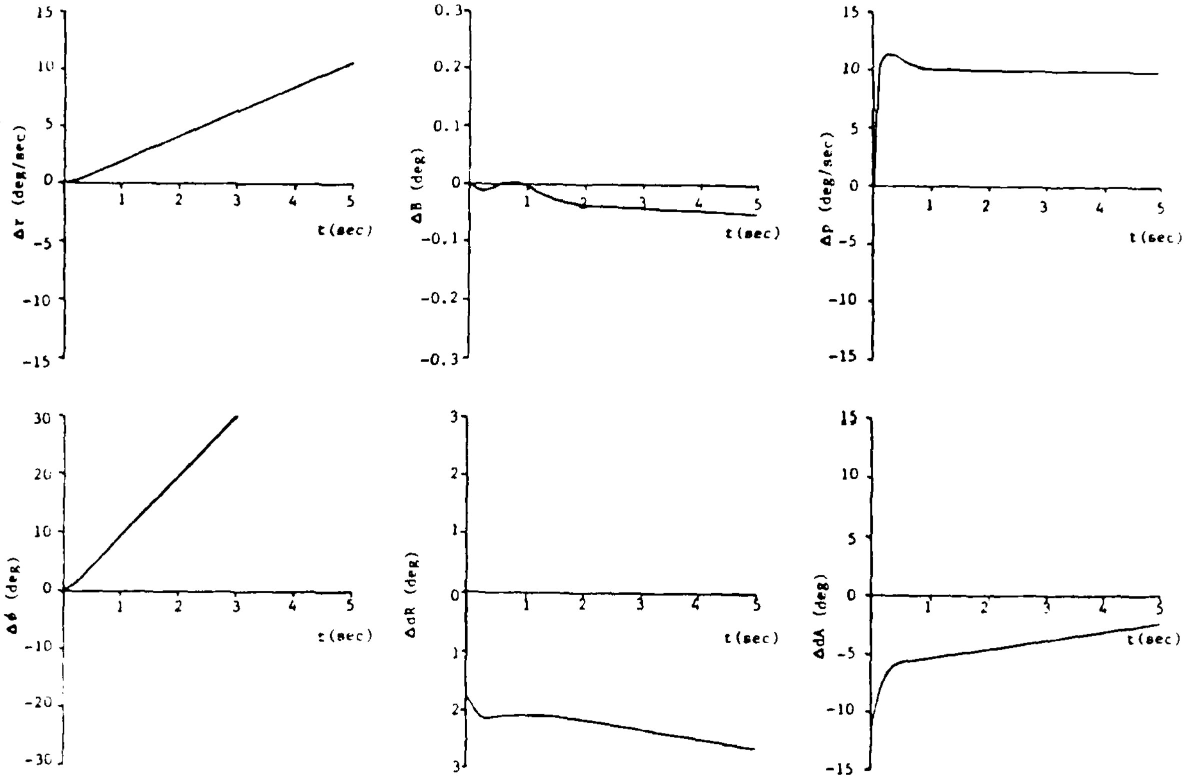


Figure 8. Gain Schedule Simulation - Roll Rate Command

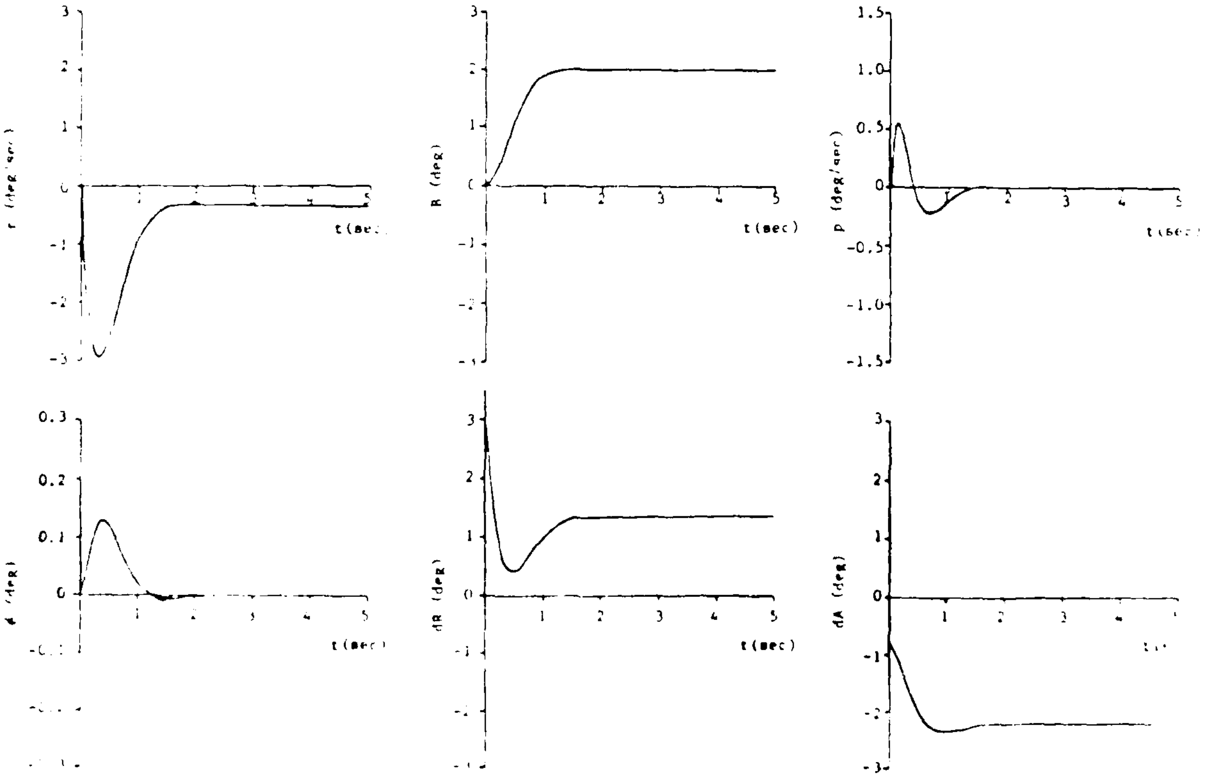
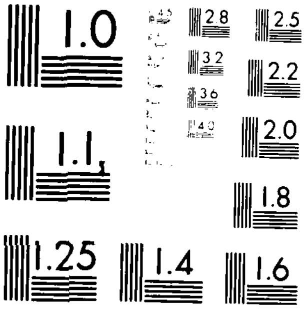


Figure 9: Gain Schedule Simulation - Sideslip Command

AD-A128 579 A LATERAL-DIRECTIONAL CONTROLLER FOR 2/3
HIGH-ANGLE-OF-ATTACK FLIGHT(U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH W A EHRENSTROM MAR 83
UNCLASSIFIED AFIT/CI/NR-83-12T F/G 1/2 NL


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

Chapter V

FLIGHT TESTING

The final test of the CAS designed in this study was to use the CAS in actual flight. To do this, CAS software compatible with the microprocessor systems was developed, implementing the control law developed in Chapter 3 and the gain schedules calculated in Chapter 4. Microprocessor limitations in memory space and computational speed as well as control system requirements of sampling time and accuracy were considered in the software development.

Once the software was developed, a limited number of ground tests were performed before the actual flight tests. The ground tests were used to check the CAS software against known simulation results and to insure that the correct signs on the outputs were generated in an actual run-time situation.

Finally, actual flight tests were performed. The flight tests included tests on the basic airframe to get an understanding of the lateral-directional characteristics in the stall regime and to help verify the model; tests of the pilot's ability to control the aircraft during the stalled conditions for comparison with the CAS; and finally, tests of the CAS operation wherein the control system was required to maintain a wings-level attitude throughout the stall maneuver.

This chapter, then describes the Microprocessor-based Digital Flight Control System (Micro-DFCS) and the software that was developed to implement the control systen. In addition, a discussion of the ground tests is included. Finally, the actual flight test results are covered including the flight test procedures, the results of the individual set of tests, and an analysis of the overall results.

5.1 DESCRIPTION OF THE MICRO-DFCS

There were four Micro-DFCS functions: accept analog inputs of aircraft states and pilot commands; update the gains; compute the control law; and output commands to the control surfaces. The microprocessor needs certain characteristics to perform these functions. It needs to have a reasonably fast computation time capability. The bit precision should be at least as good as that available from the A/D converters. Finally, the A/D and D/A converters must have a resolution compatible with aircraft sensors.

The Micro-DFCS is built around a Monolithic Systems Corporation (MSC) 8004 board. The MSC 8004 board has 32K of random-access memory (RAM), of which 28K is available for the CAS software; a programmable read-only memory (PROM) containing the Uniform Moniter (UFM) for loading, running, and dumping the CAS program; a Zilog Z80 central processing unit (CPU); and an AM-9511 high-speed mathematics unit. In addition, the Micro-DFCS has A/D and D/A converter boards. All three boards are put into a card cage and are connected to a hand-held control-display unit (CDU).

The ARA is equipped with analog, digital, and mechanical control systems. An overview of the control system interrelationship with the aircraft and pilots is presented in Figure 10. The analog and digital control systems can operate simultaneously such that digital control of the lateral-directional mode can be accomplished without affecting the longitudinal mode.

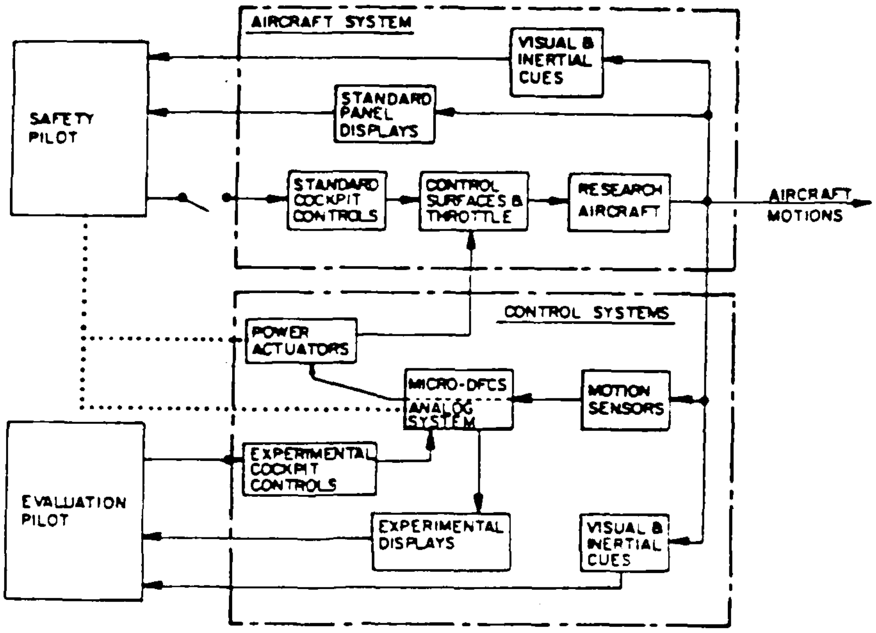


Figure 10: Overview of Aircraft Systems Configuration

5.2 CAS SOFTWARE

The CAS software was required to perform the Micro-DFCS functions for control of the aircraft. The control system required that all tasks in the control sequence (including the gain updates and the control law calculations) be accomplished within the sampling time. Because the microprocessor had limitations concerning the number of computations that could be done within any specific time period, the CAS software had to compromise these conflicting requirements.

The software was developed using the MSC 6009 disk system available at FRL. The 6009 system consisted of a MSC 6009 board, a card cage, two SMS floppy-disk drives, an ADM-31 terminal, and an Anadex 9501 line printer. The 6009 board, card cage, and disk drives are mounted, with a power supply, in a cabinet which also houses FRL's Telemetry Monitering system. The 6009 computer uses the CP/M system monitor. The monitor can be configured to run with 32K or 64K of RAM, 32K provided by the 6009 board and the additional 32K provided by other circuit boards.

There were two problems with implementation that had to be resolved for the control system to work. The first was the integration of the roll rate command to multiply by the integral gain matrix, C_i. This was easily resolved by noting that the integral of roll rate is the roll angle so that the integral could be found by the following equation:

$$\Delta p_c = \Delta \phi_c + \Delta p_c * \Delta t \qquad (5-1)$$

$$\Delta \phi_c = \Delta p_c$$

where Δp_c is the roll rate command and Δt is the sampling time. The second problem was the determination of the trim condition about which the perturbations would be measured. This was resolved by allowing the operator to set a flag to reset the nominal condition anytime the aircraft was at a new trim. The input was put in as part of the background program which ran when the control law sequence was not running.

The control sequence of CAS was initiated by an interrupt from the clock at each sampling time. When the control sequence was not being executed, a background routine accepted inputs from the CLU and had the capability of performing a limited number of tasks.

The computation of the control law began by entering the current values of the states. The perturbation values then were found by subtracting the nominal state values (i.e., the trim condition, which was set when the program was initialized) from the current value of the state. The rudder and aileron commands then were computed with the perturbations and the control gains. Finally, the commands were sent to the control surfaces.

The sequence described above did not present a problem with computation time. However, it was desirable to update the gains continuously to account for changes in the flight condition. The number of calculations required for such a task proved to be time consuming indeed. The control law calculations and update of the 14 gains required 0.25 seconds -- two and a half times the sampling interval. To run at this rate would seriously degrade the control law effectiveness, particularly since the gains were all computed with a sampling interval of 0.1 seconds.

To circumvent this problem, the gains were updated over a number of control cycles such that only one gain per sampling interval was updated. Thus, each gain was updated every 1.4 seconds. Updating more than one gain caused the control sequence to use more than its allotted time. Figure 11 shows the typical execution of the CAS program. The low break corresponds to entering the control sequence, while the high break corresponds to leaving it. As can be seen in the figure, the control sequence used about 0.06 seconds in this form.

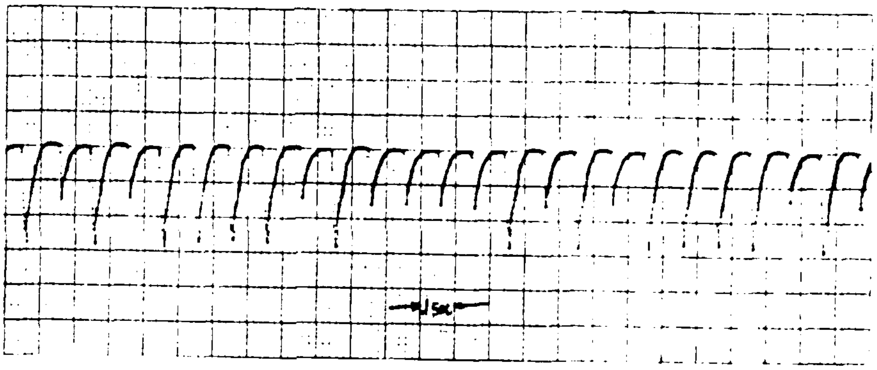


Figure 11: Control System Execution Cycle

The gain updates were accomplished prior to computation of the control law so that the most recent gains would be used. Rudder gains were updated first, followed by aileron gains. A flowchart for the control sequence is depicted in Figure 12 .

The background routine allowed the pilot to input commands for a limited number of options including: reinitialization, halting, and breaking the program execution; a test for A/D and D/A functions; and a reset for the nominal condition state values. A detailed description of the background routine and the control sequence is presented in **Appendix C**.

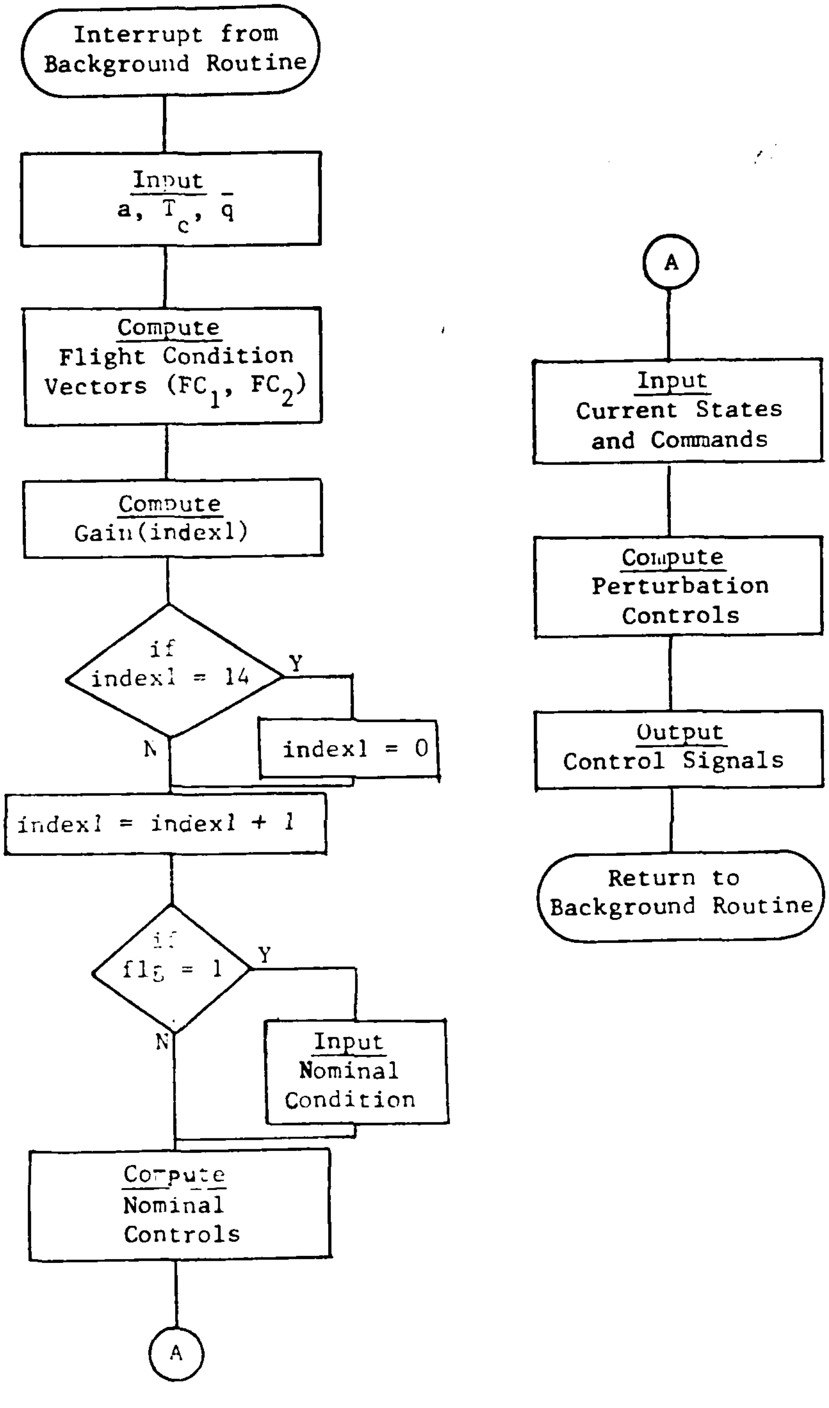


Figure 12: Flowchart of the Control Sequence

5.3 GROUND TESTS

Before the flight tests were performed, a preliminary set of tests was accomplished on the ground to insure that the software computations were correct and that the correct signals were being sent to the control surfaces. Two sets of ground tests were used to verify the CAS operation.

The first test included interfacing the microprocessor to the analog computer and sending voltages from the analog to the microprocessor. The voltages corresponded to known values of the state and flight condition variables found from simulations. Outputs were then checked to insure that the correct voltages were being sent out from the microprocessor to the control surfaces. To facilitate this check, the software had a ground test mode which enabled each step of the control law calculation to be sent to the line printer. These results were then cross-checked against simulation results and the discrepencies corrected.

Once it was ascertained that the control system calculations were being done correctly, a second set of ground tests was performed to insure that the signs of the outputs were correct. The aircraft control surfaces were capable of being operated on the ground using auxiliary power and hydraulic sources. The CAS was loaded onto the airplane as it would be during the actual flight tests and then inputs were generated for the microprocessor using the aircraft sensors. The tests were used to prove that for a given input, certain control surface deflections would be generated. With the completion of these tests, the control system was ready for actual flight situations.

5.4 FLIGHT TESTS

The objective of the flight tests was to examine the capabilities of the CAS into the stall regime and compare the results to those of the pilot in a similar situation. It was hoped that the control system would maintain lateral-directional stability better than the pilot could and thus free the pilot to perform other tasks. To accomplish this objective, a series of flight tests were developed and flown to examine different aspects of the aircraft, pilot, and control system.

The first set of tests was set up to examine the stability of the basic aircraft in the stall regime. To do this, the lateral-directional controls were locked in a trim condition just prior to stalling the aircraft. Based upon the results of the model developed in Chapter 2, the aircraft was expected to exhibit lateral-directional instabilities once the stall was encountered. The results of this set of tests pointed out the problems that the pilot and CAS were required to overcome.

The second set of tests looked at the pilot's abilities during the stall. In this case, the pilot attempted to maintain a wings-level attitude while the aircraft was stalled. Based on previous stall testing, the pilot was expected to have trouble overcoming the instabilities. The results from these tests were used in comparison with the CAS operation.

The final set of tests examined the control system capabilities. In these tests, the pilot stalled the aircraft while the CAS attempted to maintain lateral-directional trim. From the results of the CAS development, the control system was expected to provide stability for the aircraft into the stall regime. These tests were used to determine the success or failure of the CAS based on its ability to maintain stability.

For each of these tests, a couple of conditions were introduced which were known to affect the stability of the aircraft. Since it had been found that throttle setting affected stability (Ref. 1), two throttle settings were examined. In addition, "pitch pumping", or the rapid oscillation of the angle of attack, was also found to radically affect lateral-directional stability. In this case, it was also interesting to find out how well the CAS could keep up with the oscillations since the gains were angle-of-attack sensitive but were rescheduled completely only once every 1.4 seconds. Thus, each set of tests included four test runs including: low power setting with no pitch pumping; low power with pitch pumping; high power with no pitch pumping; and, finally, high power with pitch pumping. The tests outlined above are summarized in Table 7.

Documentation of the test results for analysis and inclusion into the report was accomplished using a data telemetry system already incorporated into the aircraft and ground station. In this case, the aircraft telemetry system received data on the aircraft attitude from aircraft sensors. These data then were transmitted to a recording system at the ground station.

Table 7

FLIGHT TEST DOCUMENTATION

Trim Conditions: V=70 kts., $\delta f=0°$, MAP=20", RPM=2500,
Mixture=Normal

Run	Lat/Dir Controls	Power	Pitch Pulse
1-1	Locked	15"/2500 RPM	N
1-2			Y
1-3		25"/2500 RPM	N
1-4			Y
2-1	Manual	15"/2500 RPM	N
2-2			Y
2-3		25"/2500 RPM	N
2-4			Y
3-1	CAS	15"/2500 RPM	N
3-2			Y
3-3		25"/2500 RPM	N
3-4			Y

For these tests, the data which could be recorded were limited to four channels. The flight condition variables, angle of attack and velocity, were recorded to ascertain the onset and severity of the stall. In addition, these two variables (plus throttle seting which was constant for each test run) were used to schedule the gains. Along with these variables, two lateral-directional attitude variables were recorded: sideslip and roll angle. These variables were used to determine the performance of the pilot and CAS during the test runs.

5.4.1 Airframe Tests

The purpose of these tests was to get a feel for the stability of the basic airframe in a stalled condition. To accomplish these tests, the aircraft's lateral-directional controls were trimmed and locked into position. The aircraft was then stalled and its response recorded. From there, the data were analyzed and a picture of the lateral-directional stall characteristics was formed.

From the model, a preliminary idea of how the aircraft should react in a stall was obtained. The eigenvalues for the aircraft at a stall angle of attack, low power setting and low airspeed point to an unstable roll-spiral mode. In this case, a divergent roll angle with some oscillations was expected. For high power setting, the eigenvalues predict the same type of response with a slightly longer time constant and slower oscillation rate. No information was available from the model pertaining to the effect of pitch pumping.

Figures 13 through 16 present the results of this set of tests. In each test, the stall is characterized by the aircraft exceeding the critical angle of attack, which from NASA TN D-5758 was found to be 18 deg angle of attack. (The spikes which occur throughout the data were the result of telemetry dropouts.)

In the case of the low power settings (Figures 13 and 14), the results were as expected. In each case, as the stall was encountered, the aircraft began to oscillate around a slowly divergent roll angle. In Figure 13, with no pitch pumping, the aircraft was trimmed for a roll angle of 3.2 deg and a sidesl p of 4.5 deg. Twenty seconds after the stall was encountered, the roll angle reached a maximum of 61.5 deg before the aircraft was recovered. Sidesl p, while apparently not divergent, was very erratic, ranging from -9.5 deg to 5.8 deg -- as much as 14.2 deg from trim. The maximum angle of attack encountered was 33.1 deg and the minimum velocity was 61.6 knots.

In Figure 14, low power setting with pitch pumping, the results were similar. The trim conditions were 4.8 deg for roll angle and 7.7 deg for sideslip. The maximum roll angle prior to aircraft recovery was 69.4 deg occurring 28 sec after the stall was encountered. Sideslip ranged from 0.0 deg to 11.9 deg and as much as 7.7 deg from trim. The maximum angle of attack was 34.7 deg. During pitch pumping, the aircraft was subjected to changes in angle of attack as high as 15.6 deg/sec. The minimum velocity encountered was 61.6 knots.

The problem of stalling at a higher power setting is shown in Figures 15 and 16. In both cases, the aircraft showed very little oscillation compared to the low power setting but the divergences were of larger magnitude with higher roll rates.

In Figure 15, with no pitch pumping, the aircraft was trimmed with a roll angle of 6.5 deg and a sideslip of 7.1 deg. Once the stall was encountered, the aircraft remained somewhat stable for a few seconds as the angle of attack increased. Sideslip departed trim first, oscillating from -16.5 deg then back to 18.7 deg. Maximum divergence from trim was 23.6 deg. Roll angle hesitated prior to departing the trim condition and then simply rolled onto one wing. Maximum roll angle was undetermined since the plot went off the scale but was in excess of 75.0 deg. Large roll rates as high as 36.5 deg/sec were encountered. Maximum angle of attack was also undetermined but was in excess of 40.0 deg. The minimum velocity was 50.0 knots.

In Figure 16, high power setting with pitch pumping, the results are similar. The aircraft was timmmed for a roll angle of 6.5 deg and a sideslip of 7.1 deg. After the aircraft was stalled, sideslip once again departed trim first, ranging from 15.2 deg to -17.1 deg -- as much as 24.2 deg from trim. Roll angle shows the aircraft rolling one direction and then back the other before being recovered 21 sec after the stall was encountered. Maximum roll angle was 79.9 deg and was as low as -37.9 deg. The maximum change in angle of attack was 18.2 deg/sec. Minimum velocity was 50.0 knots.

In comparing the results of including pitch pumping against those where pitch pumping was not done, there seems to be little effect on the stability of the aircraft. Indeed, with the exception of the roll left in Run 1-4 prior to the roll right, the same power setting gave similar results regardless of pitch pumping.

It is also interesting to note that the aircraft always ends up rolling towards the right wing and that the maximum deviation from trim sideslip always occurs with negative sideslip. These results are due to torque effects of the engine and point to problems for both the pilot and CAS in trying to maintain stability of the aircraft into the stall regime.

Overall, the aircraft performed as predicted by the model. At the low power settings, the aircraft went into a divergent oscillatory roll once stall was encountered. At the higher power settings, less oscillation was encountered. The maximum divergence occurred around the same time in both cases.

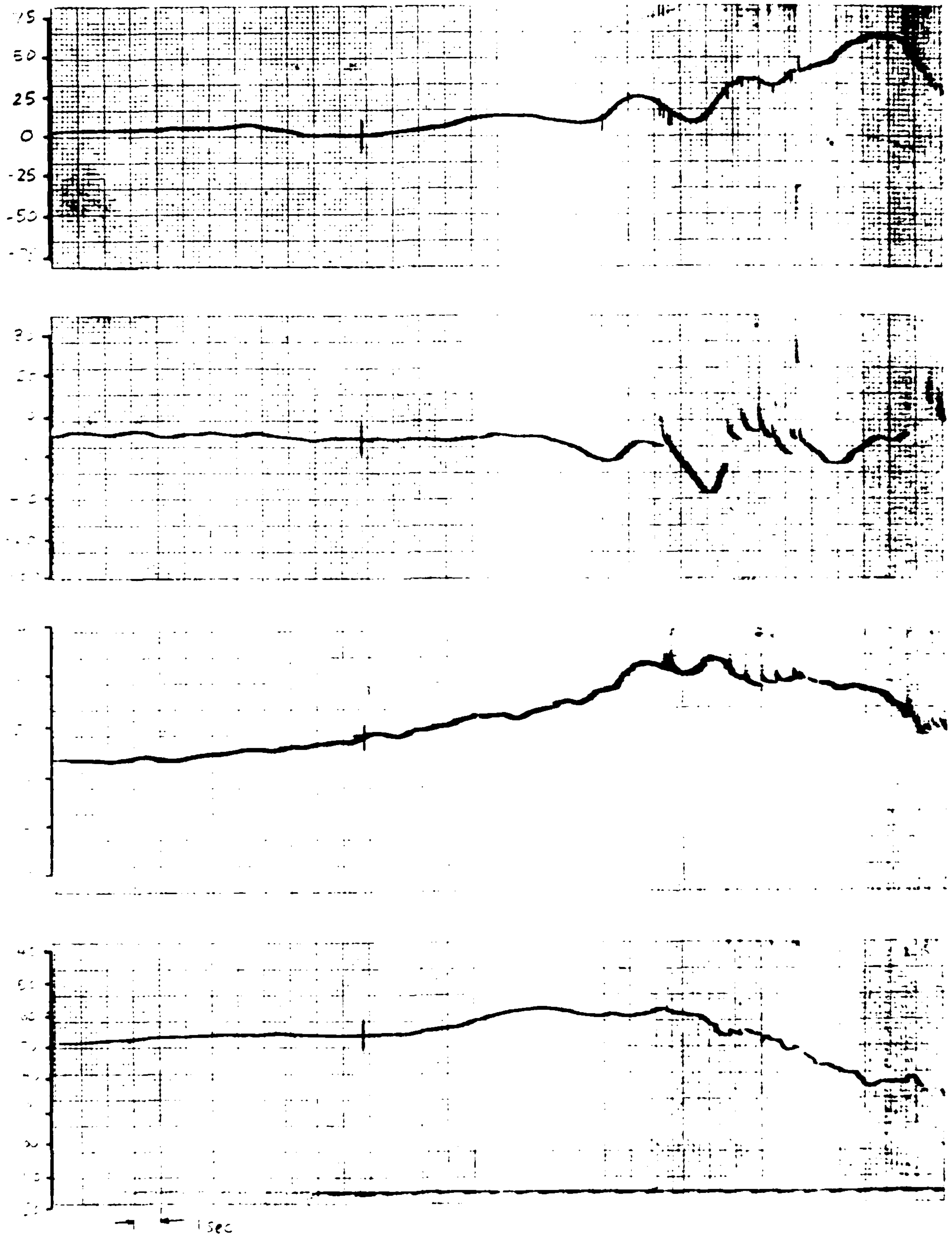


Figure 13. Flight Test Run 1-1 Results.

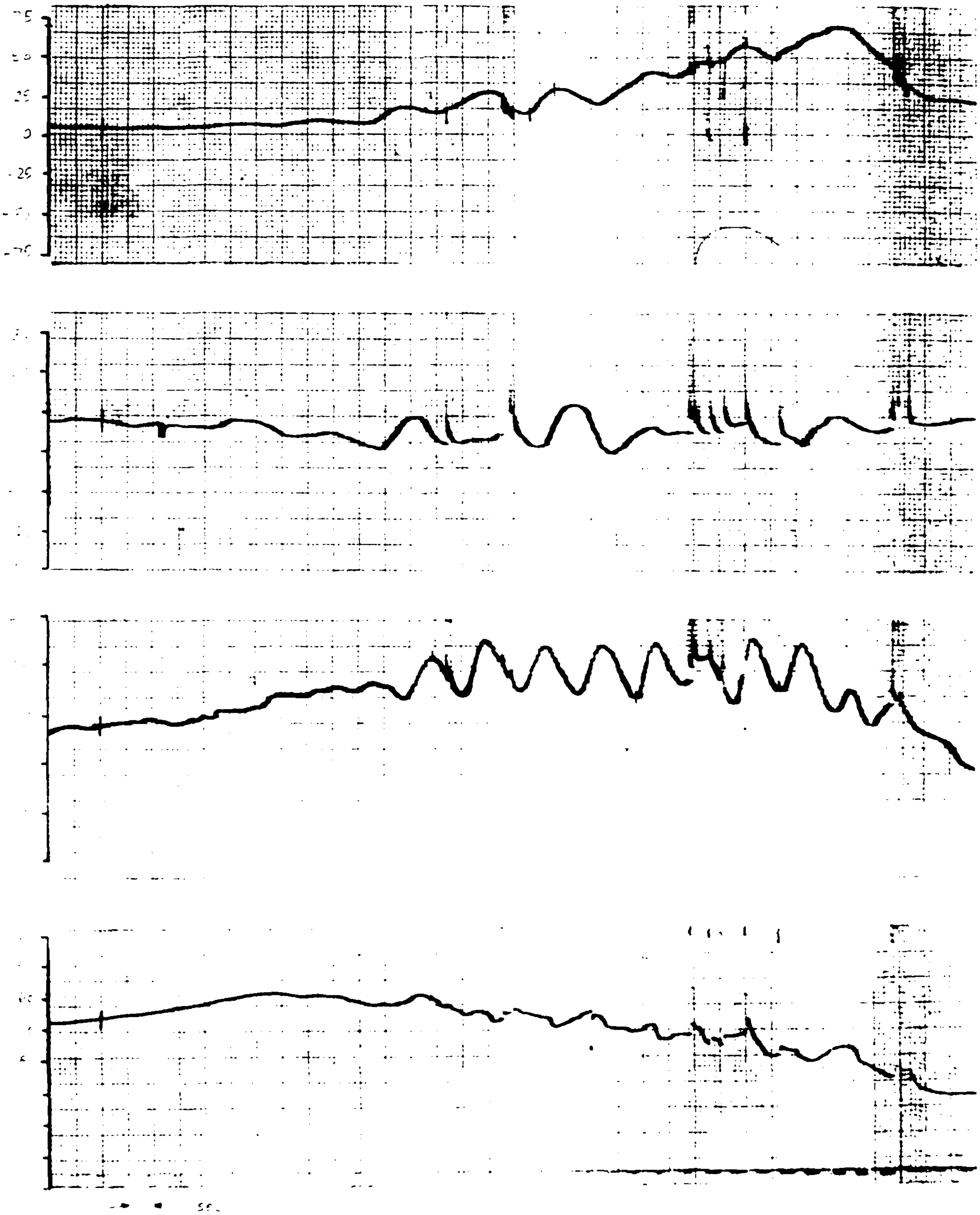


Figure 14. Flight Test Run 1-2 Results.

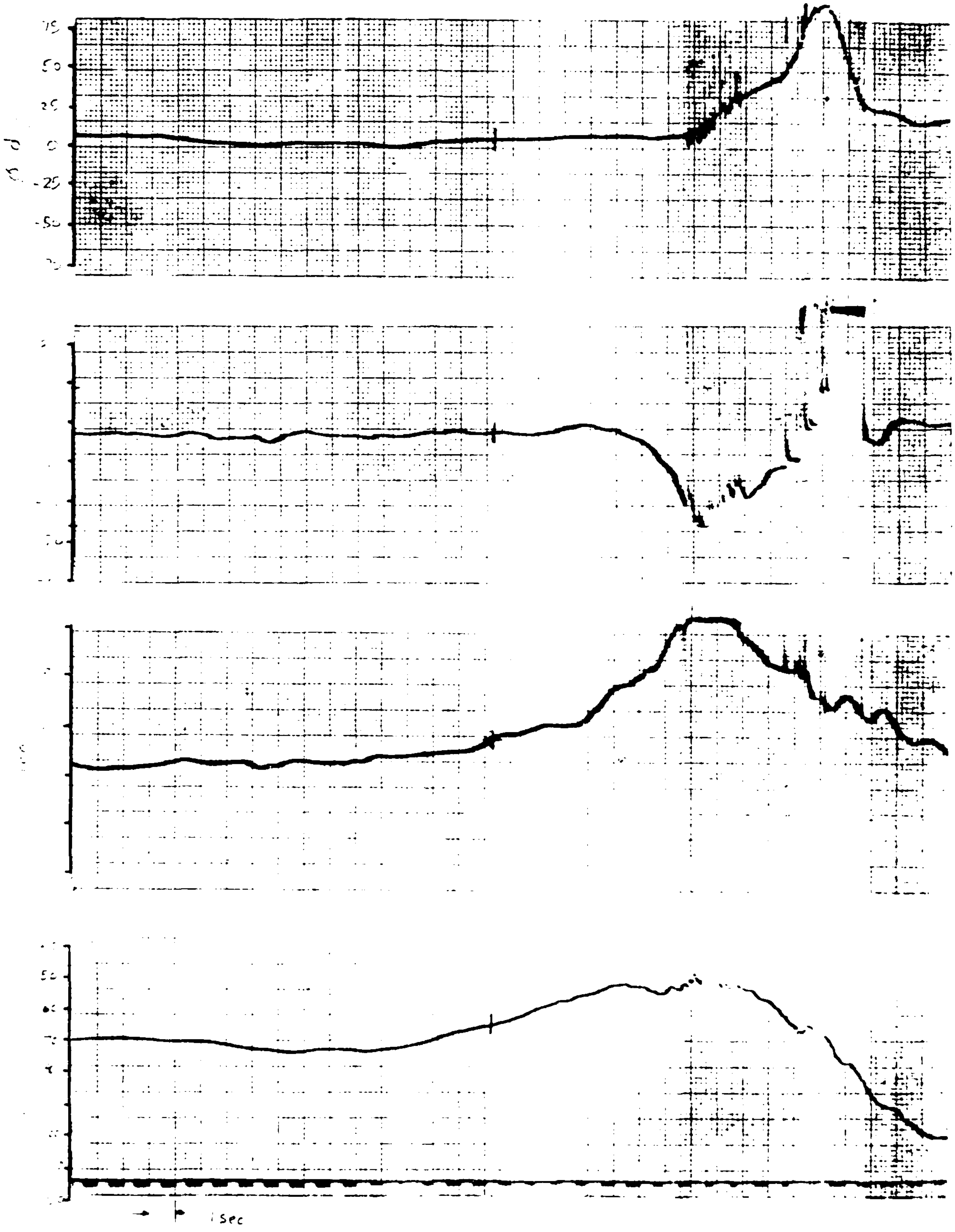


Figure 15. Flight Test Run 1-3 Results.

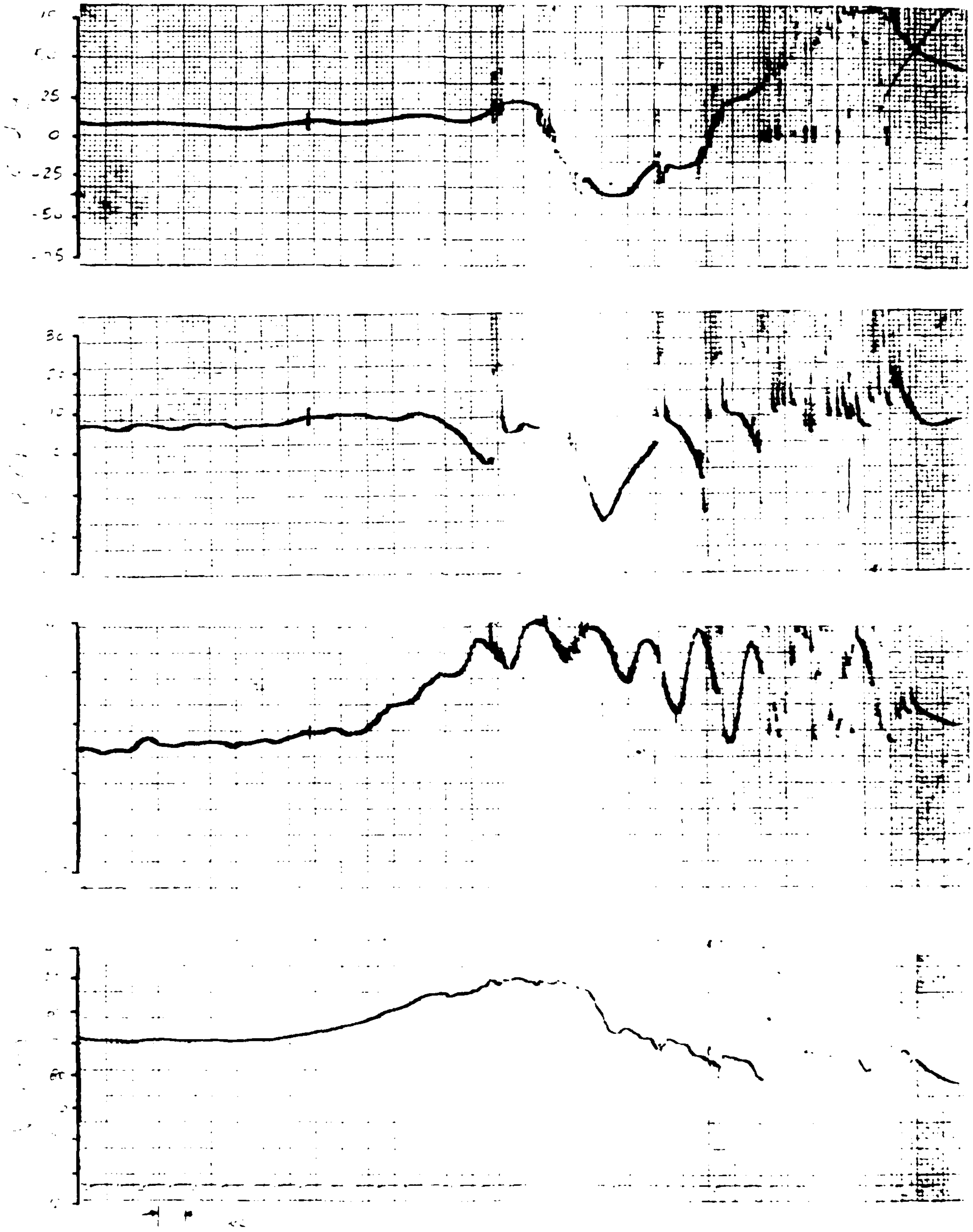


Figure 16. Flight Test Run 1-4 Results.

5.4.2 Pilot Tests

The purpose of this set of tests was to determine how well the pilot was able to maintain lateral-directional stability when the aircraft was stalled. The results were used as a baseline for comparison with the CAS in a similar situation. Figures 17 through 20 present the results.

At low power settings (Figures 17 and 18), the pilot was able to control the instabilities fairly well. In Figure 17, low power setting without pitch pumping, the aircraft's lateral-directional attitude showed only minor deviations. Roll angle ranged from 4.9 deg to 12.9 deg and was in general pretty steady. Sideslip was also fairly steady, ranging from 1.3 deg to 9.4 deg. The maximum angle of attack encountered was 32.6 deg and the minimum velocity was 53.2 knots.

Figure 18 shows the results of low power setting with pitch pumping. Once again, the pilot was able to control the instabilities well into the stall regime, though the addition of pitch pumping caused larger deviations. Roll angle varied anywhere from -1.6 deg to 33.1 deg while sideslip varied from 0.0 deg to 16.8 deg. The maximum angle of attack encountered was 37.8 deg with rates of change as high as 15.6 deg/sec. Minimum velocity was 53.2 knots.

The higher power settings proved to be much more difficult for the pilot. Both roll angle and sideslip deviations became large compared to low power settings. In particular, the sideslip deviations became unmanageable once the stall was encountered.

Figure 19 shows the results of high power setting without pitch pumping. Roll angle deviations were large though manageable, varying from -9.7 deg to 26.6 deg. Sideslip variations were also large and much more erratic. Sideslip angles from -9.4 deg to 12.6 deg were encountered. Maximum angle of attack was 37.9 deg while minimum velocity was 51.6 knots.

Figure 20 presents the results of high power setting with pitch pumping. In this case, the aircraft's instabilities became unmanageable for the pilot. Roll angle varied anywhere from -34.7 deg to 43.5 deg and would have continued to increase had the aircraft not been recovered. Sideslip, too, showed large deviations ranging from -25.8 deg to 20.7 deg and was continuing to increase up until aircraft recovery. The highest angle of attack was 41.0 deg, changing at reates as high as 24.3 deg/sec during pitch pumping. Minimum velocity encountered was 52.1 knots.

In comparing the results of no pitch pumping to those where pitch pumping was added, some definite differences were noted. At low power settings without pitch pumping, only small deviations in lateral-directional attitude were noted. When pitch pumping was included, the deviations became markedly larger, particularly in the case of sideslip. Total variation of roll angle without pitch pumping (measured by the angle between the minimum and maximum roll angle) was 8.0 deg compared to 34.7 deg with pitch pumping. For sideslip, total variation without pitch pumping was 8.1 deg compared to 16.8 deg with pitch pumping.

The results were similar in the case of high power setting. Total roll angle variation without pitch pumping was 36.3 deg compared to 78.2 deg with pitch pumping. Total side-slip deviation without pitch pumping was 22.0 deg compared to 46.5 deg when pitch pumping was included.

In contrast to the airframe tests, then, pitch pumping made a discernable difference in the results where the pilot was required to maintain lateral-directional stability. This could be attributed to a number of reasons. First, in trying to maintain stability at low airspeeds, the pilot was using sluggish control surfaces. Since the aircraft's attitude was changing rapidly (due to the inclusion of pitch pumping), the combination of relatively slow reaction time of the pilot and sluggish control surfaces would make it difficult to overcome the instabilities.

Second, whereas the pilot had outside references for maintaining wings level, there were no similar references for sideslip. If the pitch pumping caused sideslip deviations, the pilot would not have been able to sense and correct them. Flying in a sideslip would tend to aggravate the aircraft's instabilities.

Finally, just the effort of including pitch pumping into the results may have taken away from the pilot's attention enough that he was unable to maintain stability as well as when pitch pumping was not included.

Overall, this set of tests showed that differences in power setting and the inclusion of pitch pumping made a real difference in how well the pilot was able to maintain stability. Though no simulation results were available for comparison, the pilot did perform as had been found in previous studies.

5.4.3 CAS Tests

The final set of tests was run to determine how effective the control system designed in this study was in eliminating the instabilities encountered when the aircraft was stalled. To accomplish this task, the aircraft was trimmed prior to actuating the control system, and the aircraft was stalled. The control system then was required to maintain the lateral-directional trim into the stall.

The control system was designed with relatively rapid response times. Roll rate response was selected to be as quick as possible, while the sideslip response was chosen to be about one second. In both cases, the response times were found to be sufficient to provide stability into the stall regime in all simulation results. The results from this set of tests

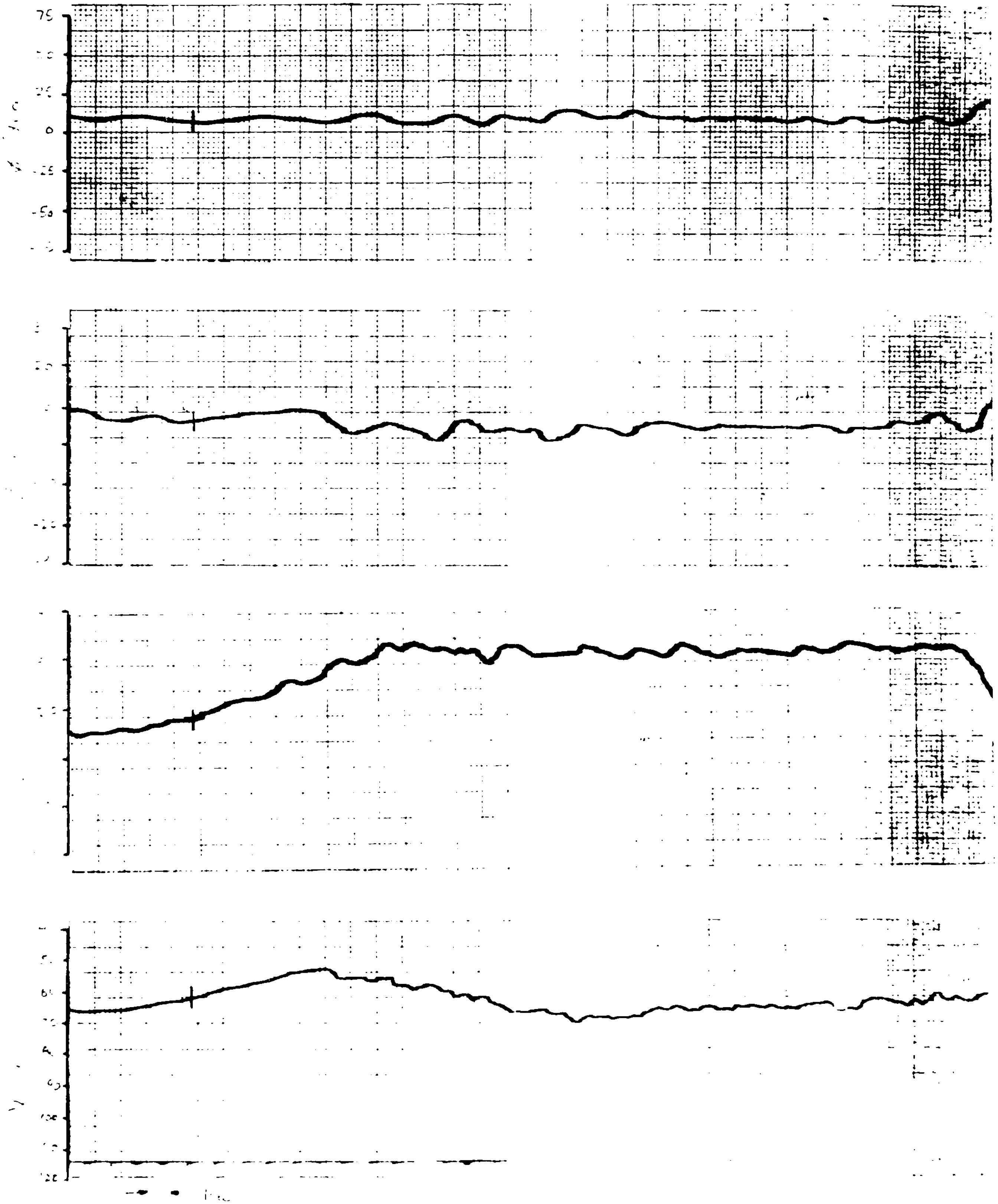


Figure 17. Flight Test Run 2-1 Results.

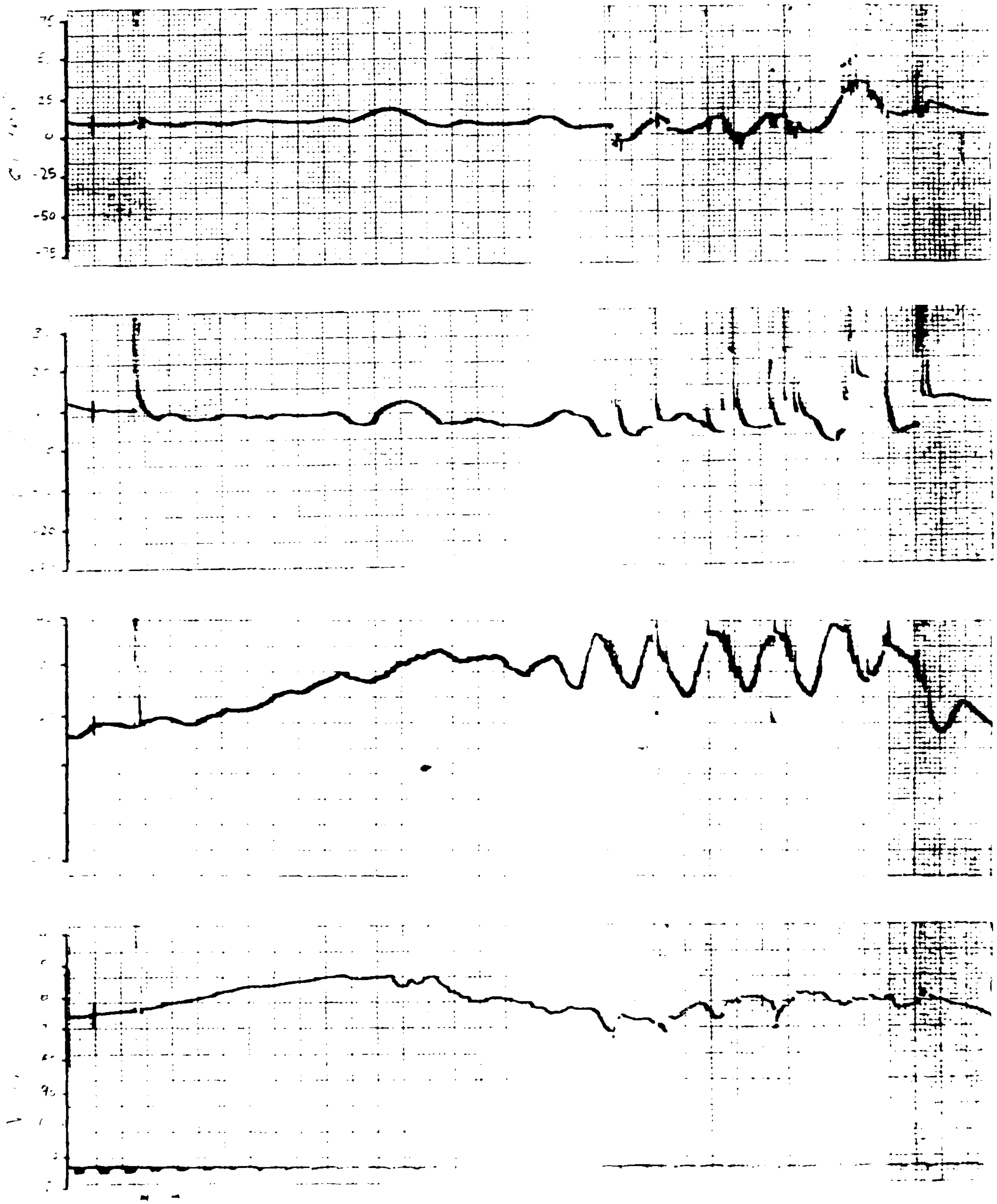


Figure 18. Flight Test Run 2-2 Results.

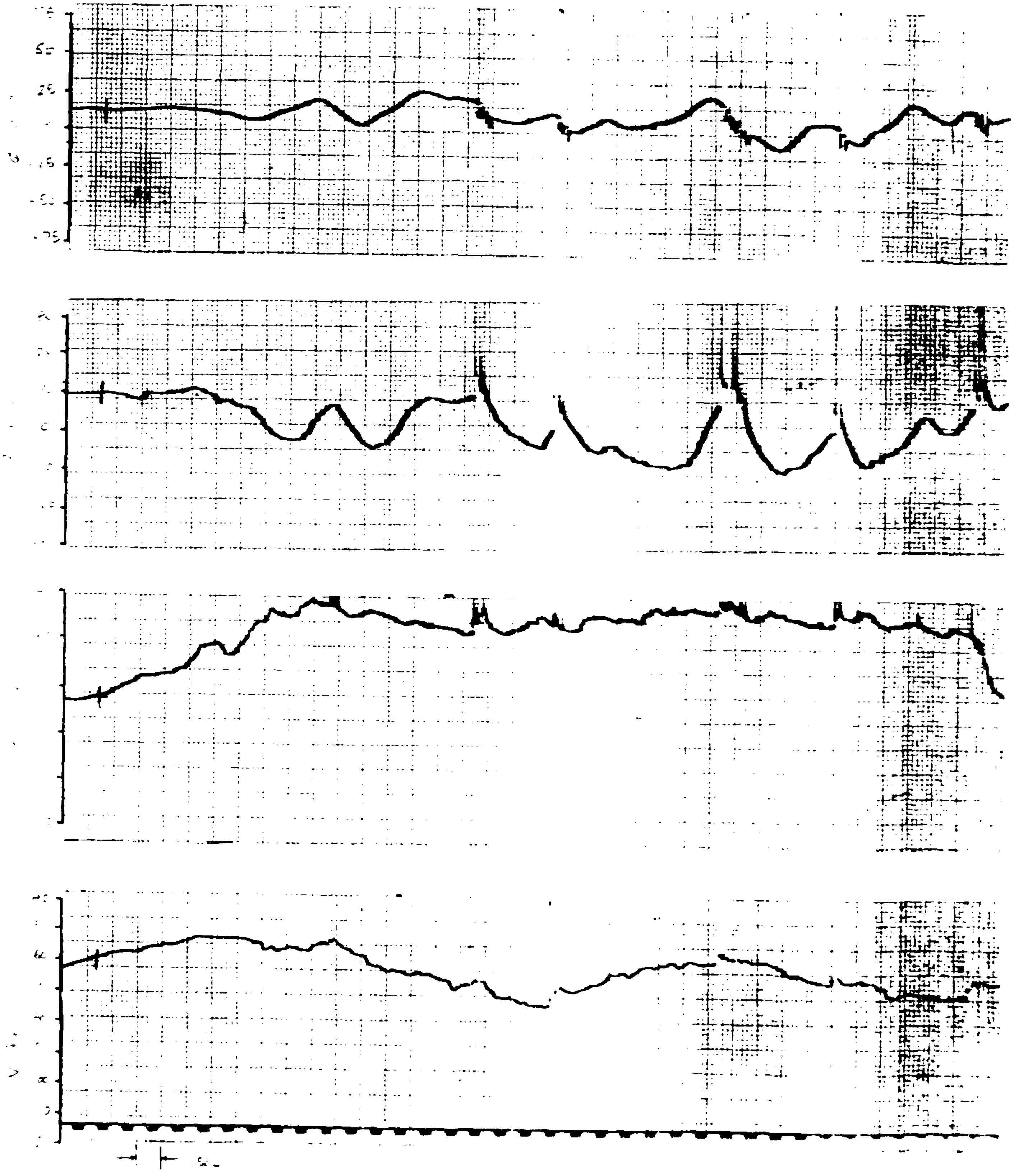


Figure 19. Flight Test Run 2-3 Results.

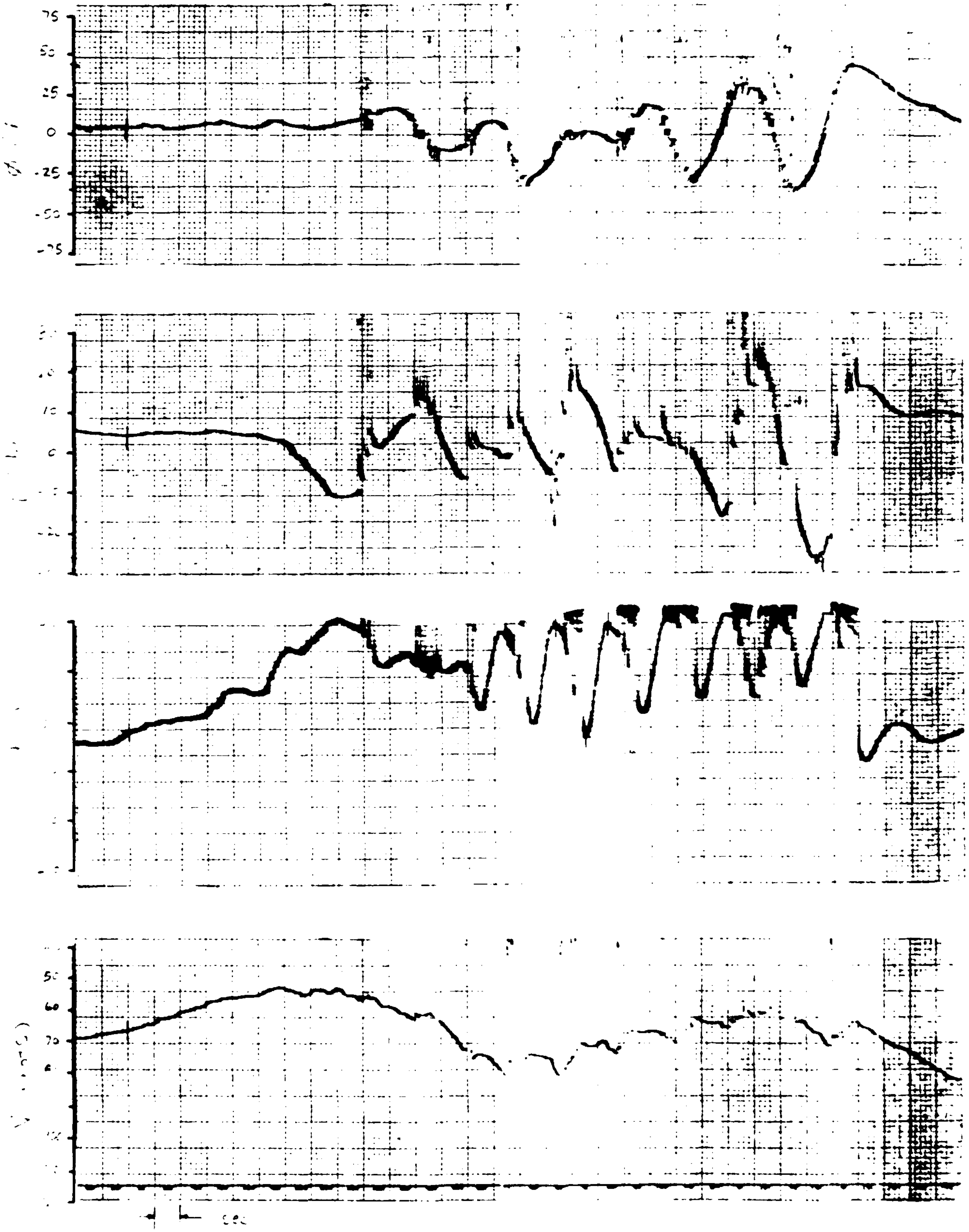


Figure 20. Flight Test Run 2-4 Results.

are presented in Figures 21 through 24.

The low power setting results are shown in Figures 21 and 22. When no pitch pumping was present (Figure 21), the CAS had no trouble at all maintaining stability. Roll angle showed negligible variation from the trim value of 8.1 deg. Sideslip did show some deviations from the 8.1 deg trim value, ranging from 8.7 deg to 3.2 deg -- as much as 4.9 deg from trim. This deviation is probably due to the slower response time demanded for sideslip. The maximum angle of attack was 30.0 deg while the minimum velocity was 50.0 knots.

When pitch pumping was included (Figure 22), the CAS still responded to maintain lateral-directional stability. Roll angle was trimmed at 4.9 deg. Once stall was encountered and the pitch pumping begun, the aircraft did a slow, counterclockwise roll to -6.5 deg. This slow roll was probably due to the inability of the control system to keep up with the rapidly changing flight condition. Sideslip also showed some minor deviations during pitch pumping. Sideslip was trimmed for 5.2 deg but was as high as 11.3 deg or 6.1 deg from trim. The maximum angle of attack was 31.0 deg while the maximum rate of change of the angle of attack was 20.8 deg/sec. The minimum velocity encountered was 51.6 knots.

Higher power settings also did not pose a problem for the control system though larger deviations from trim were noted than at the lower power settings. In Figure 23, high power setting without pitch pumping, roll angle was trimmed at 9.7 deg and varied from 16.2 deg to 6.5 deg or as much as 6.5 from trim. Sideslip was trimmed at 8.7 deg and varied from 11.3 deg to 5.2 deg or as much as 3.5 deg from trim. The maximum angle attack encountered was 30.0 deg while the minimum velocity was 50.0 knots.

When pitch pumping was included with a higher power setting (Figure 24), the results were still more than adequate. Roll angle was trimmed at 9.7 deg and varied between 16.2 deg and 3.2 deg. Maximum deviation from trim was 6.5 deg. An oscillation occurred when the pitch pumping was ended and the aircraft recovered and was probably due to the control system catching up to the current flight condition. Sideslip was trimmed at 11.3 deg and varied from 13.2 deg to 6.8 deg -- as much as 4.5 deg from trim. The largest deviations occurred during the pitch pumping phase of the test and, once again, are probably due to the inability of the control system to keep up with the changing flight condition. The maximum angle of attack was 36.9 deg while the largest rate of change for angle of attack encountered during pitch pumping was 13.4 deg/sec. Minimum velocity was 47.6 knots.

As was in the case of the pilot tests, pitch pumping did have an effect on the results, though not as drastic. At low power settings, the test without pitch pumping showed no variation in roll angle and a total sideslip variation of 5.5 deg.

When pitch pumping was included, total roll angle variation went up to 11.3 deg and total sideslip variation to 6.1 deg. The same kind of results occurred at the higher power settings as well. Total roll angle variation without pitch pumping was 9.7 deg compared to 12.9 deg with pitch pumping. Total sideslip variation without pitch pumping was 6.1 deg while with pitch pumping, it was 6.5 deg. These results were due to the slow speed at which the control system updated the gains and the fact that the angle of attack was changing at rates as high as 20.8 deg/sec. Regardless, the CAS provided adequate control throughout the stall.

One problem with the control system was encountered which makes it unusable in its present form. The aircraft's control surfaces are set up with a safety feature to cutout if the CAS commands excessively large deflections at any one time. This was a problem for the CAS since it was operating in a very unstable region where the control effects were sluggish. Large control surface deflections were required to maintain aircraft control. In the case of several test runs, numerous attempts were needed to get one "successful" run where cutouts did not occur too early.

Several possibilities exist for why this problem arose. First, when the weighting matrices were selected, a minimum roll rate response was considered desirable. However, this may force the CAS to correct deviations from trim too fast and cause excessively large aileron commands. Another possible problem is that of sampling time. At high angles of attack, the aircraft is very unstable in the roll-spiral mode. The aircraft can diverge quite rapidly, as was demonstrated in the first set of tests, so the control system must be capable of reacting equally as fast. The CAS would have to make up for a slow sampling time by using large control surface deflections. Finally, the safety limits on the aircraft may be too tight for the regime in which the control system was tested and could be widened somewhat. Thus, the solution to the cutout problem would be to make changes in all three potential problem areas -- slow down roll rate response, increase the samples per interval of time, and relax the control surface safety limits on the aircraft.

Another answer to the cutout problem might be low-pass filtering. This scheme includes not only control surface deflection into the gain computation process, but also the deflection rate. The deflection rate could be limited to an acceptable level where the control surfaces would not cutout. It could also reduce the jerkiness associated with large contron surface deflections.

Overall, the control system proved to be a success in eliminating the aircraft's instabilities encountered in the stall regime. It was able to overcome the problems of sluggish control response and slow gain updating and still provide adequate control. Control surface cutouts did detract from the performance of the system but can be remedied with some minor changes in the gain updating scheme.

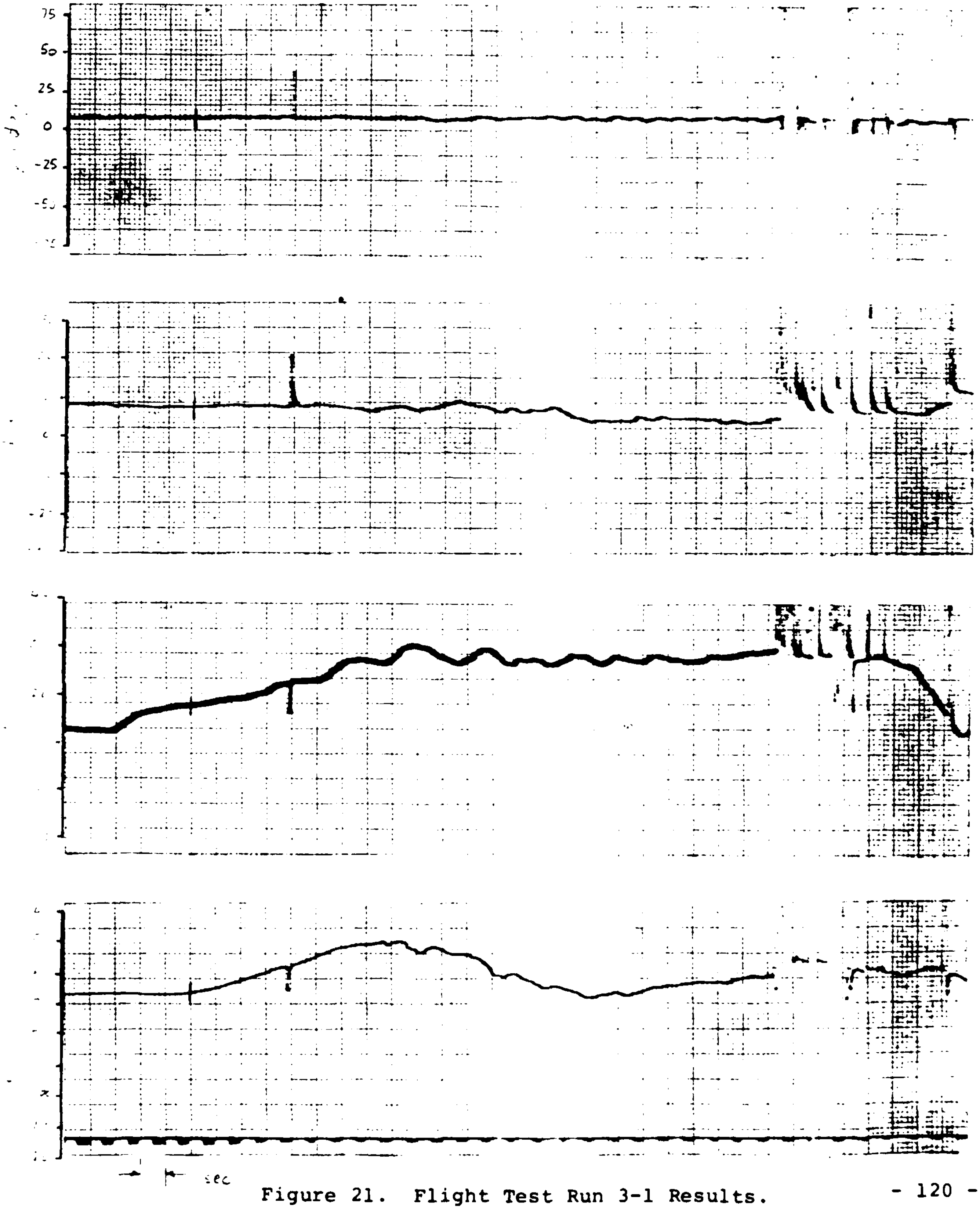


Figure 21. Flight Test Run 3-1 Results.

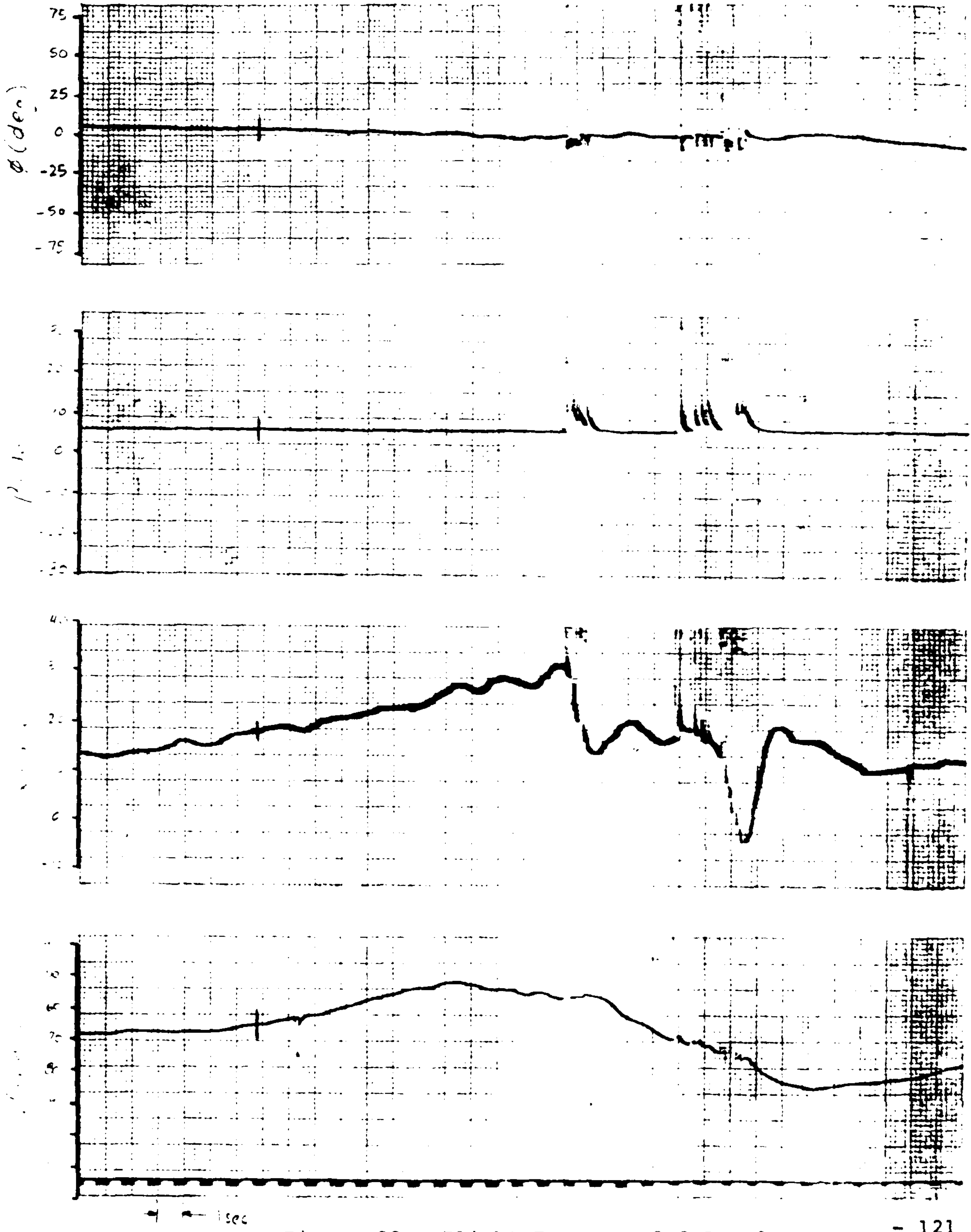


Figure 22. Flight Test Run 3-2 Results.

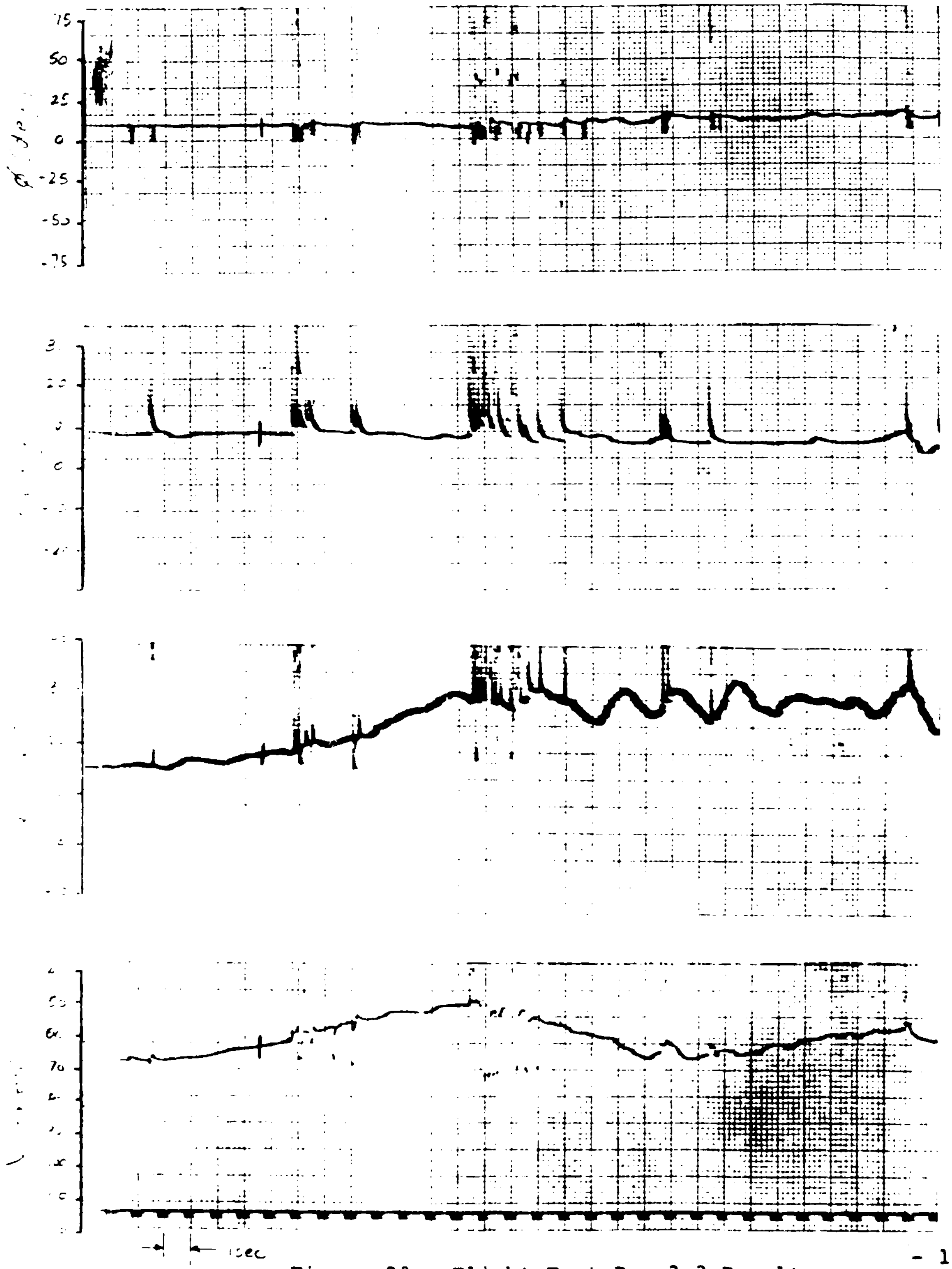


Figure 23. Flight Test Run 3-3 Results.

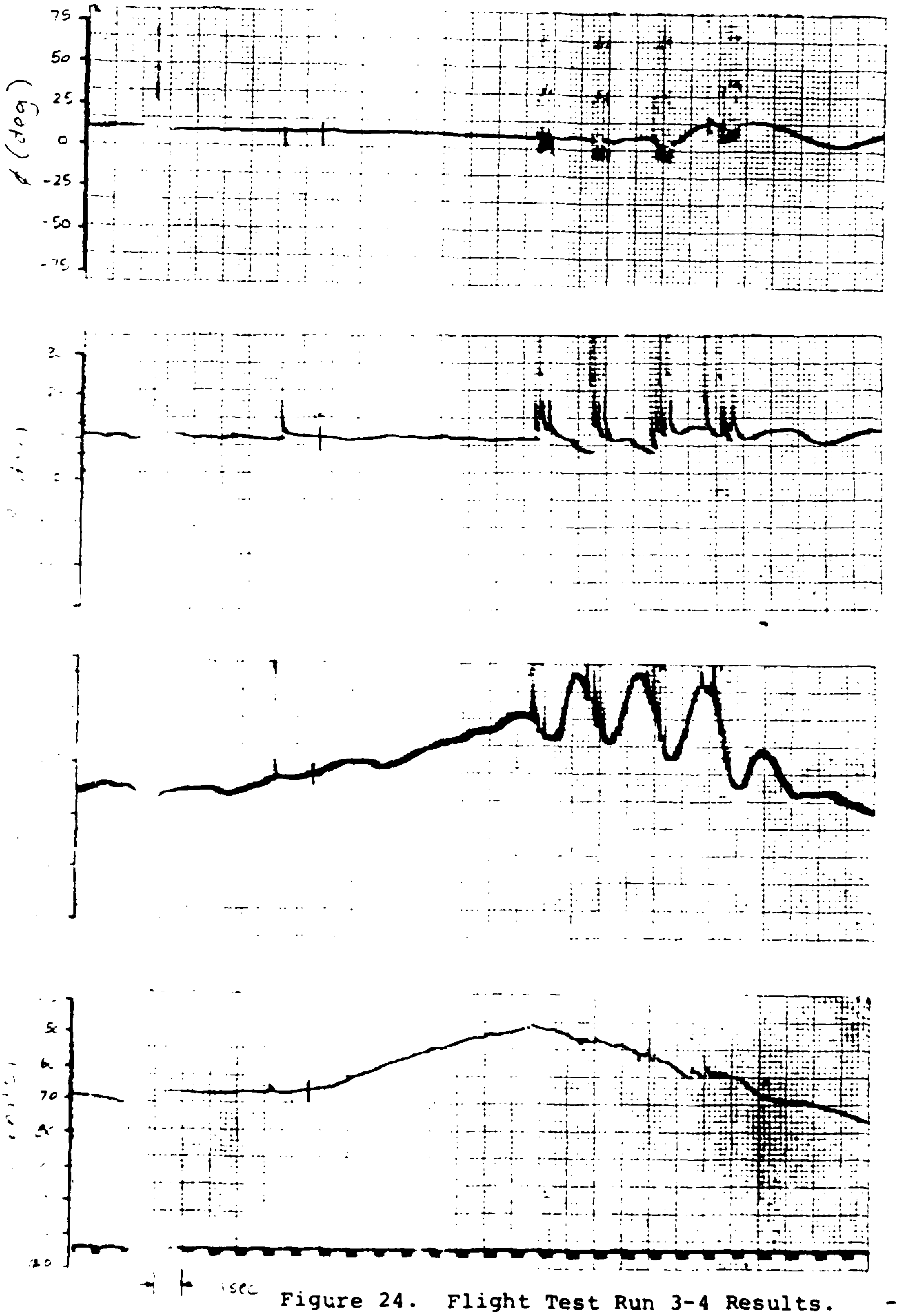


Figure 24. Flight Test Run 3-4 Results.

5.5 ANALYSIS OF RESULTS

To summarize the results, the flight tests were successful at providing an understanding of the aircraft in the stall regime and how to eliminate the instabilities encountered there. The airframe tests pointed out the problems of stalled flight while the pilot tests showed how difficult it was for the pilot to overcome the instabilities experienced in the stall. The CAS tests proved that it is indeed possible to eliminate the instabilities and provide safer and more controlled stall flight characteristics.

The primary purpose of the airframe tests was to get a better understanding of how the aircraft responds in the stall regime. Based on the model, the roll-spiral mode was found to be unstable, which would point to a divergent, oscillatory roll. This was found to be the case in the actual tests. Higher power setting was found to lessen the oscillatory portion of the roll-spiral instability but increase the speed at which the divergences occur. Low power settings caused roll rates as high as 12.9 and 16.2 deg/sec for cases without and with pitch pumping, respectively, while at higher power settings, those same maximum roll rates became 35.5 and 44.4 deg/sec, respectively.

It was also interesting to note that the aircraft always departed controlled flight in the same direction due to the torque effects of the propeller. In this case, the aircraft always ended up rolling toward the right wing and flying with negative sideslip. These observations point to problems that both the pilot and control system had to overcome.

Finally, it was found that the inclusion of pitch pumping had only a small effect on the stability of the aircraft. The more interesting effect of pitch pumping was how well the pilot and CAS were able to handle the aircraft when pitch pumping was included.

The purpose of the pilot and CAS tests was to determine how well the control system was able to help the pilot control the unstable aircraft. When the results of the tests were compared, the CAS did provide a much larger degree of stability than the pilot in the same situation. Marked reductions in the total variation are found in both roll angle and sideslip.

In the case of low power setting with no pitch pumping, the pilot had a total roll angle variation of 8.1 deg compared to no variation for the control system. For sideslip, the pilot allowed 8.1 deg total variation while the CAS allowed only 5.5 deg. When pitch pumping was included, the pilot had a total roll angle variation of 34.7 deg compared to only 11.3 deg for the CAS. Total pilot sideslip variations, in this case, were 16.8 deg compared to only 6.1 deg for the control system.

The comparisons for the high power setting are even more striking. Without pitch pumping, the pilot allowed a total roll angle variation of 36.3 deg compared with only 9.7 deg total variation allowed by the control system. Total sideslip variation for the pilot was 22.0, while the CAS had only 6.1 deg total variation. With pitch pumping, the pilot had total roll angle variation of 78.2 deg compared with 12.9 deg for the CAS. The total sideslip variation for the pilot was 46.5 deg compared to 6.5 deg for the control system. Also at the high power settings, the pilot results showed that the variations would have continued to increase had the aircraft not been recovered,while the control system results show no such tendency. It was readily apparent, then, that the control system could be an invaluable aid to the pilot in controlling an otherwise unstable aircraft.

One major problem was discovered with the control system implementation after the tests had been run. The control system was designed around the stability axes of the aircraft. However, the aircraft sensors, from which the aircraft attitude was measured and sent to the computer, are mounted on the aircraft and are not angle-of-attack sensitive. No accomodations were made for this fact within the control system software. Thus the control system utilized stability-axis software with body-axis inputs. To remedy this problem, the sensor inputs should be transformed to stability axes, based on the current angle of attack, prior to the computation of the control surface outputs.

In addition, there were some minor problems noted in the actual flight tests. In particular, the cutout problem previously mentioned needs to be worked out. Some jerkiness also was experienced in the control surface actuations due to the large control surface deflections and the relatively long sampling time. One last problem was that the control system had some trouble keeping up with a rapidly changing flight condition. It is possible that these problems may be the result of the modelling or control law implementation errors. They may also point to some fine tuning required in the control law.

It should be pointed out that regardless of the errors encountered in this project, the control system was still a success and did accomplish its goal. Overall, the flight tests proved that it is possible to provide stability to an otherwise unstable aircraft, even into the stall regime. Such a control system could provide additional safety for pilots, particularly unskilled pilots, when operating in the regions near stall.

Chapter VI

CONCLUSIONS AND RECOMMENDATIONS

The study of aircraft in the high-angle-of-attack regime can provide great rewards in terms of increasing the knowledge of flight and subsequently enhancing aircraft safety. However, that same regime many challenging problems which must be overcome before the rewards can be attained. By putting together a test platform to investigate the different aspects of stalled flight, it would be possible to overcome the problems and make important steps toward improving flight safety.

The purpose of this study was to design and test a control system to eliminate the lateral-directional disturbances from the longitudinal mode in stalled flight. At high angles of attack, the lateral-directional mode becomes unstable and difficult for even an experienced pilot to overcome. By designing a control system to provide stability, even into the stall regime, it would be possible to isolate the longitudinal mode and take steps toward improving aircraft safety.

Perhaps the most challenging part of the problem was that the aircraft could be stalled at an infinite number of flight conditions. Designing a CAS that only provided control for one flight condition would be useless at best. Therefore, the control system was designed for a whole range of flight conditions where the gains were made a function of the flight condition.

Thus, a model of the aircraft had to be computed to represent the aircraft at any flight condition.

The model was assembled using wind tunnel data for the aircraft found in NASA TN D-5758. For those terms for which data were not available, the USAF DATCOM calculation method was used. The data were reduced to a number of equations where the stability, rotary, and control derivatives were calculated from the current flight condition. This method proved to be an effective one. The results of the simulations compare well with actual results of airframe flight tests as explained in Chapter 5. The model then was used to derive control gains for the CAS.

The CAS was designed as a sampled-data, linear-quadratic controller. A number of sample flight conditions were chosen for which suitable control gains were calculated. Simulations showed satisfactory control of the aircraft at each of the sample flight conditions. The gains then were scheduled as functions of the flight condition. These scheduled gains were then tested in simulation and were also found to provide adequate control for each flight condition provided that a minimum correlation was maintained between the actual gains and the scheduled gains.

Finally, operational control system software was formulated for the purpose of flight tests. The flight tests were broken down into three parts. The first was a test of the basic airframe to get a better understanding of the lateral-directional mode in a stall. The results of the tests pointed out the instabilities that the pilot and CAS were required to overcome. The results also verified the model since the aircraft reacted as predicted from simulation results.

The second part of the flight tests included pilot tests to see how well the pilot could handle the instabilities in a stall. The results were also used as a baseline for comparison with the CAS results. The tests showed that the pilot could maintain some control at lower power settings but that the control was lost at higher power settings.

The final part of the flight tests was the test of the CAS during a stall. The results show that the control system was able to maintain stability well into the stall regime. Deviations from trim were present but were not near the magnitude of those encountered with the pilot in control. Therefore, the study was a success at proving that it is indeed possible to provide stability for an otherwise unstable aircraft, even into the stall regime. The ramifications for this type of CAS point to improved aircraft safety, especially for unskilled pilots, not only in the high-angle-of-attack and stall regimes, but also throughout the whole range of flight conditions the aircraft may encounter.

In reexamining the project, errors were found in the modelling and control law implementation. In particular, the angle-of-attack effects on the inertia matrix and the vertical component of velocity were neglected and resulted in an erroneous model. This, in turn, affected the computation of the control gains. In addition, no provision was made for converting raw sensor data to the stability-axis system in the computer software. These problems would have to be remedied before the control system could be used effectively. It should be noted, however, that even with these errors, the control system did accomplish its goal of stabilizing the aircraft in a stall.

Further work is needed to make some improvements in the control system to make it more workable. In particular, a more efficient gain scheduling scheme would be desirable to reduce the size of the gain matrices while retaining the needed accuracy. This, in turn, would allow a reduction in the sampling time with several added benefits: elimination of cutouts, smoothing of the jerkiness, and better response during rapid changes in the flight condition. A look into the area of proportional filtering might also eliminate large control surface deflections and their associated problems.

In conclusion, then, this study has shown that it is indeed possible to improve lateral-directional stability into the stall regime of the aircraft. Such a find promises to enhance the study of high-angle-of-attack flight. It also provides a starting point for the design of safety systems into the aircraft to help eliminate instabilities experienced during high-angle-of-attack flight.

Appendix A

AVIONICS RESEARCH AIRCRAFT

A.1 DESCRIPTION OF THE AIRCRAFT

The aircraft used in this study is the Avionics Research Aircraft (ARA), a modified Navion. The ARA is capable of three modes of control -- direct, analog, and digital -- which can operate simultaneously for a particular application. In addition, the aircraft has been equipped with inertial, air data, and navigation sensors. The aircraft has been used in stall-spin research as well as control system design.

The sensors which are available for telemetry and control system usage include angular rate gyros and linear accelerometers for all three axes, angle-of-attack and sideslip vanes, an airspeed sensor, and control position indicators. In addition, barometric altitude, airspeed, air temperature, and engine manifold pressure also are available to the pilot.

The aircraft is flown with two pilots for reasons of flight test efficiency and safety. The evaluation pilot sits in the left seat which is set up for fly-by-wire control system operation for both digital and analog applications. For analog control systems, pilot inputs are routed through potentiometers to the control surfaces. The potentiometers are located on the display panel and can be adjusted in flight to vary the aircraft's handling characteristics. For digital control systems, a microprocessor is used. The computer itself is located behind the right seat and is interfaced with the pilot through the hand-held CDU. The safety pilot sits in the right seat and has direct control over the control surfaces through the use of mechanical linkages.

A.2 AIRCRAFT DATA

A Navion airframe was tested extensively in the 30' x 60' full-scale wind tunnel at the NASA Research Center, Hampton, Virginia and the results of those tests were compiled in the report, NASA TN D-5857. The availability of the data simplified the model development process. To use it, the data were reduced to a set of tables of data points from the nondimensional coefficient curves. The tables are presented here and are arranged as follows: Table 8, constant component data; Table 9, stability derivative component data; Table 10, longitudinal coefficient data; Table 11, rudder derivative component data; and Table 12, aileron derivative component data.

In addition, a number of aircraft constants were required for the rotary derivative component development. Those constants are presented in Table 13.

TABLE 8

Constant Component Data

a η_c	0.03	0.12	0.23
-4.0	0.001	0.002	0.007
0.0	0.0	0.0	0.0
4.0	0.0	-.004	-.007
8.0	-.002	-.008	-.016
12.0	-.005	-.015	-.028
16.0	-.019	-.025	-.044
20.0	-.018	-.040	-.065
24.0	-.020	-.045	-.090

C_y

a η_c	0.03	0.12	0.23
-4.0	0.0010	0.0	-.0040
0.0	0.0005	-.0015	-.0060
4.0	-.0005	-.0030	-.0080
8.0	-.0010	-.0035	-.0090
12.0	-.0015	-.0020	-.0090
16.0	0.0020	0.0	-.0050
20.0	0.0050	0.0015	0.0030
24.0	0.0020	0.0070	0.0100

C_n

a η_c	0.03	0.12	0.23
-4.0	0.0110	0.0090	0.0020
0.0	0.0090	0.0065	0.0060
4.0	0.0090	0.0055	0.0040
8.0	0.0100	0.0050	0.0020
12.0	0.0070	0.0050	0.0025
16.0	-.0030	0.0030	0.0030
20.0	0.0090	0.0065	-.0025
24.0	0.0120	0.0020	0.0100

C_l

TABLE 9

Stability Derivative Component Data

(throttle setting = 0.03)

a ^{E}	-15.0	-10.0	-5.0	0.0	5.0	10.0	15.0
-4.0	0.182	0.134	0.075	0.001	-.066	-.148	-.210
0.0	0.177	0.131	0.069	0.0	-.070	-.145	-.206
4.0	0.170	0.110	0.062	0.0	-.066	-.142	-.202
8.0	0.160	0.100	0.055	-.002	-.060	-.135	-.197
12.0	0.143	0.085	0.044	-.005	-.050	-.110	-.185
16.0	0.121	0.075	0.030	-.019	-.063	-.104	-.171
20.0	0.102	0.050	0.010	-.018	-.054	-.097	-.155
24.0	0.084	0.050	0.005	-.020	-.050	-.097	-.130

c_y

a ^{E}	-15.0	-10.0	-5.0	0.0	5.0	10.0	15.0
-4.0	-.0275	-.0210	-.0120	0.0010	0.0135	0.0265	0.0365
0.0	-.0260	-.0200	-.0115	0.0005	0.0130	0.0220	0.0325
4.0	-.0250	-.0180	-.0110	-.0005	0.0105	0.0175	0.0290
8.0	-.0230	-.0160	-.0105	-.0010	0.0075	0.0140	0.0275
12.0	-.0210	-.0135	-.0095	-.0015	0.0030	0.0120	0.0220
16.0	-.0165	-.0110	-.0085	0.0020	0.0040	0.0120	0.0195
20.0	-.0185	-.0080	-.0010	0.0050	0.0105	0.0135	0.0240
24.0	-.0235	-.0150	-.0020	0.0020	0.0100	0.0105	0.0160

c_n

a ^{E}	-15.0	-10.0	-5.0	0.0	5.0	10.0	15.0
-4.0	0.0330	0.0280	0.0210	0.0110	-.0050	-.0110	-.0170
0.0	0.0330	0.0240	0.0190	0.0090	-.0025	-.0080	-.0120
4.0	0.0270	0.0225	0.0170	0.0090	-.0040	-.0100	-.0115
8.0	0.0220	0.0220	0.0160	0.0100	-.0070	-.0140	-.0150
12.0	0.0250	0.0230	0.0150	0.0070	-.0120	-.0200	-.0215
16.0	0.0300	0.0190	0.0140	-.0030	-.0175	-.0260	-.0290
20.0	0.0440	0.0370	0.0330	0.0090	-.0055	-.0275	-.0325
24.0	0.0500	0.0430	0.0200	0.0120	-.0060	-.0320	-.0360

c_l

Table 9

continued

(throttle setting = 0.23)

a B	-15.0	-10.0	-5.0	0.0	5.0	10.0	15.0
-4.0	0.243	0.180	0.106	0.008	-.080	-.160	-.237
0.0	0.230	0.168	0.095	0.0	-.090	-.167	-.246
4.0	0.213	0.155	0.082	-.008	-.099	-.175	-.252
8.0	0.197	0.142	0.068	-.016	-.104	-.185	-.256
12.0	0.180	0.127	0.053	-.029	-.104	-.190	-.257
16.0	0.158	0.104	0.037	-.043	-.096	-.162	-.250
20.0	0.133	0.070	0.005	-.065	-.110	-.158	-.221
24.0	0.093	0.004	-.050	-.100	-.108	-.128	-.228

C_y

a B	-15.0	-10.0	-5.0	0.0	5.0	10.0	15.0
-4.0	-.0320	-.0250	-.0190	-.0040	0.0100	0.0200	0.0330
0.0	-.0320	-.0275	-.0205	-.0060	0.0075	0.0190	0.0305
4.0	-.0320	-.0280	-.0220	-.0075	0.0055	0.0175	0.0285
8.0	-.0315	-.0270	-.0235	-.0090	0.0035	0.0160	0.0265
12.0	-.0295	-.0240	-.0240	-.0090	0.0015	0.0140	0.0250
16.0	-.0260	-.0200	-.0220	-.0050	0.0	0.0105	0.0240
20.0	-.0215	-.0130	-.0110	0.0030	0.0040	0.0140	0.0225
24.0	-.0160	-.0015	0.0080	0.0140	0.0105	0.0180	0.0215

C_n

a B	-15.0	-10.0	-5.0	0.0	5.0	10.0	15.0
-4.0	0.0385	0.0290	0.0260	0.0020	-.0130	-.0130	-.0170
0.0	0.0330	0.0265	0.0215	0.0000	-.0100	-.0120	-.0130
4.0	0.0280	0.0230	0.0175	0.0040	-.0085	-.0115	-.0130
8.0	0.0245	0.0190	0.0130	0.0020	-.0080	-.0130	-.0160
12.0	0.0220	0.0170	0.0085	0.0025	-.0095	-.0160	-.0200
16.0	0.0260	0.0195	0.0120	0.0030	-.0140	-.0235	-.0240
20.0	0.0350	0.0220	0.0145	-.0025	-.0210	-.0240	-.0390
24.0	0.0400	0.0265	0.0220	0.0000	-.0250	-.0160	-.0240

C_l

TABLE 10

Longitudinal Coefficient Data

a T_c	0.03	0.12	0.23
-4.0	-.180	-.180	-.140
0.0	0.140	0.150	0.210
4.0	0.455	0.480	0.550
8.0	0.750	0.800	0.895
12.0	1.040	1.120	1.240
16.0	1.270	1.365	1.525
20.0	1.300	1.500	1.550
24.0	1.370	1.380	1.620

C_L

a T_c	0.03	0.12	0.23
-4.0	0.015	-.070	-.190
0.0	0.020	-.065	-.190
4.0	0.025	-.060	-.180
8.0	0.040	-.040	-.160
12.0	0.070	0.0	-.115
16.0	0.130	0.055	-.060
20.0	0.270	0.140	0.075
24.0	0.375	0.420	0.280

C_D

TABLE 11

Rudder Derivative Component Data

(throttle setting = 0.03)

a dR	-17.5	-9.0	0.0	7.0	13.2
-4.0	-.050	-.020	0.002	0.018	0.044
0.0	-.054	-.020	0.0	0.018	0.050
4.0	-.058	-.024	-.005	0.014	0.050
8.0	-.061	-.027	-.006	0.011	0.047
12.0	-.063	-.031	-.006	0.008	0.040
16.0	-.076	-.036	-.017	0.003	0.030
20.0	-.077	-.050	-.015	-.004	0.015
24.0	-.060	-.028	-.015	0.002	0.047

C_y

a dR	-17.5	-9.0	0.0	7.0	13.2
-4.0	0.0280	0.0130	0.0010	-.0060	-.0210
0.0	0.0300	0.0125	0.0	-.0080	-.0255
4.0	0.0315	0.0125	-.0005	-.0095	-.0280
8.0	0.0320	0.0130	-.0010	-.0100	-.0290
12.0	0.0300	0.0145	-.0010	-.0085	-.0250
16.0	0.0390	0.0140	0.0020	-.0060	-.0200
20.0	0.0350	0.0195	0.0055	-.0015	-.0130
24.0	0.0250	0.0090	0.0015	-.0060	-.0150

C_n

a dR	-17.5	-9.0	0.0	7.0	13.2
-4.0	0.0010	0.0070	0.0110	0.0080	0.0135
0.0	0.0065	0.0040	0.0085	0.0115	0.0145
4.0	0.0065	0.0075	0.0095	0.0135	0.0155
8.0	0.0100	0.0080	0.0100	0.0090	0.0130
12.0	0.0020	0.0065	0.0075	0.0080	0.0060
16.0	0.0035	0.0040	-.0035	-.0015	-.0040
20.0	0.0175	0.0090	0.0090	0.0080	0.0045
24.0	0.0195	0.0120	0.0115	0.0135	0.0035

C_l

Table 11

continued

(throttle setting = 0.23)

a dR	-17.5	-9.0	0.0	7.0	13.2
-4.0	-.063	-.018	0.000	0.033	0.070
0.0	-.084	-.033	0.0	0.027	0.061
4.0	-.100	-.040	-.006	0.020	0.055
8.0	-.115	-.058	-.018	0.011	0.050
12.0	-.120	-.068	-.030	0.003	0.046
16.0	-.140	-.074	-.044	0.003	0.044
20.0	-.130	-.100	-.065	-.031	0.015
24.0	-.140	-.080	-.090	-.080	-.086

C_y

a dR	-17.5	-9.0	0.0	7.0	13.2
-4.0	0.0365	0.0115	-.0040	-.0160	-.0375
0.0	0.0400	0.0110	-.0060	-.0210	-.0430
4.0	0.0425	0.0115	-.0070	-.0230	-.0470
8.0	0.0445	0.0120	-.0080	-.0250	-.0500
12.0	0.0440	0.0130	-.0080	-.0255	-.0505
16.0	0.0475	0.0160	-.0055	-.0200	-.0495
20.0	0.0420	0.0230	0.0040	-.0150	-.0370
24.0	0.0395	0.0170	0.0100	0.0025	-.0170

C_n

a dR	-17.5	-9.0	0.0	7.0	13.2
-4.0	-.0040	0.0020	0.0020	0.0125	0.0230
0.0	-.0010	-.0010	0.0060	0.0090	0.0190
4.0	0.0	-.0015	0.0035	0.0050	0.0190
8.0	0.0020	0.0	0.0020	0.0055	0.0140
12.0	0.0055	0.0065	0.0010	0.0060	0.0070
16.0	0.0085	0.0075	0.0020	0.0015	-.0030
20.0	0.0140	0.0010	-.0025	-.0015	-.0055
24.0	0.0055	0.0085	0.0065	-.0030	-.0060

C_l

TABLE 12

Aileron Derivative Component Data

a dA	-42.0	-21.0	0.0	21.0	42.0
-4.0	0.015	0.012	0.005	0.0	-.010
0.0	0.012	0.010	0.0	-.002	-.010
4.0	0.010	0.010	0.0	-.005	-.011
8.0	0.006	0.008	-.005	-.010	-.015
12.0	0.007	0.018	-.006	-.015	-.024
16.0	-.007	0.0	-.015	-.027	-.040
20.0	0.005	-.018	-.018	-.022	-.053
24.0	-.015	-.015	-.015	-.022	0.020

C_y

a dA	-42.0	-21.0	0.0	21.0	42.0
-4.0	0.0050	0.0010	0.0010	0.0	-.0020
0.0	0.0010	0.0	0.0	0.0100	0.0
4.0	-.0025	-.0030	0.0	0.0015	0.0025
8.0	-.0065	-.0060	-.0005	0.0030	0.0055
12.0	-.0110	-.0130	-.0010	0.0065	0.0105
16.0	-.0095	-.0110	0.0020	0.0110	0.0200
20.0	-.0155	-.0020	0.0050	0.0145	0.0285
24.0	-.0100	-.0080	0.0020	0.0130	0.0190

C_n

a dA	-42.0	-21.0	0.0	21.0	42.0
-4.0	0.0600	0.0400	0.0105	- 0170	-.0450
0.0	0.0595	0.0390	0.0090	-.0150	-.0470
4.0	0.0645	0.0385	0.0100	-.0140	-.0480
8.0	0.0645	0.0425	0.0100	-.0150	-.0480
12.0	0.0630	0.0320	0.0070	-.0180	-.0515
16.0	0.0575	0.0320	-.0030	-.0290	-.0605
20.0	0.0730	0.0390	0.0100	-.0120	-.0535
24.0	0.0620	0.0540	0.0130	-.0090	-.0200

C_l

TABLE 13

Aircraft Constants

Wing Data

Aspect Ratio	AR	6.04
Area	S_w, S	184 feet^2
Span	b_w	33.38 feet
Chord	c_w	5.7 feet
Centerline distance from c.g.	z_w	1 foot
Taper Ratio	λ_w	0.54
Dihedral	Γ_w	7.5°
Sweep (at quarter chord)	Λ_w	3.0°

Horizontal Tail Data

Aspect Ratio	AR_h	4.0
Area	S_h	43 feet^2
Span	b_h	13.11 feet
Chord	c_h	3.28 feet
Centerline distance from c.g.	z_h	0.0 feet
Taper ratio	λ_h	0.67
Sweep (at quarter chord)	Λ_h	6.0°

Vertical Tail Data

Aspect Ratio	AR_v	2.0
Area	S_v	12.5 feet^2
Span	b_v	5.0 feet
Chord	c_v	3.52 feet
Taper Ratio	λ_v	0.557
Sweep (at quarter chord)	Λ_v	20.0°

Table 13

continued

Mass and Inertia Data

Gross Weight	m	2940 lbs.
Center of Gravity (c.g.)	x/c_w	0.25
Moment of Inertia (x axis)	I_x	1284.1 slug-ft^2
Moment of Inertia (y axis)	I_y	2772.9 slug-ft^2
Moment of Inertia (z axis)	I_z	3234.7 slug-ft^2

Miscellaneous Data

Distance from Moment Center to Vertical Tail c.p. (normal to body ℄)	z_p	2.87 feet
Distance from Moment Center to Vertical Tail c.p. (parallel to body ℄)	l_p	17.29 feet
Distance of Vertical Tail Center of Pressure Above or Below Moment Center	z	$z_p*\cos(a)+l_p*\sin(a)$

Appendix B

GAIN COMPUTATION SOFTWARE

The gain computations were done by coding, in FORTRAN, the model and control law development steps into the program, **CONTRL**. **CONTRL** had the following capabilities: compute the linearized system equations given the flight condition; compute the optimal gains given a set of state control weightings; compute the equivalent closed-loop system equations; and perform a linear and/or nonlinear simulation. A flow chart of **CONTRL** is presented in Figure 13 .

In addition, a number of subroutines were included in the main program to perform intermediate tasks. **LDDYN** computed the lateral-directional dynamic equations given the flight condition and the current value of the states and controls. **FANDG** computed the linearized system and output matrices given the flight condition and the trim values of the states and controls. **STM** computed the state transition matrix given the sample time and the system dynamic matrix. **CEM** computed the control effects matrix given the state transition matrix and the system matrices. **CLOOPF** computed the equivalent closed-loop system dynamics matrix given the closed-loop state transition matrix. **RKINT** performed a nonlinear simulation using a 4-th order Runga-Kutta integration given the flight condition, the control gain matrices, and the initial condition. Table 14 presents a listing of each of these subroutines as well as the main program, CONTRL.

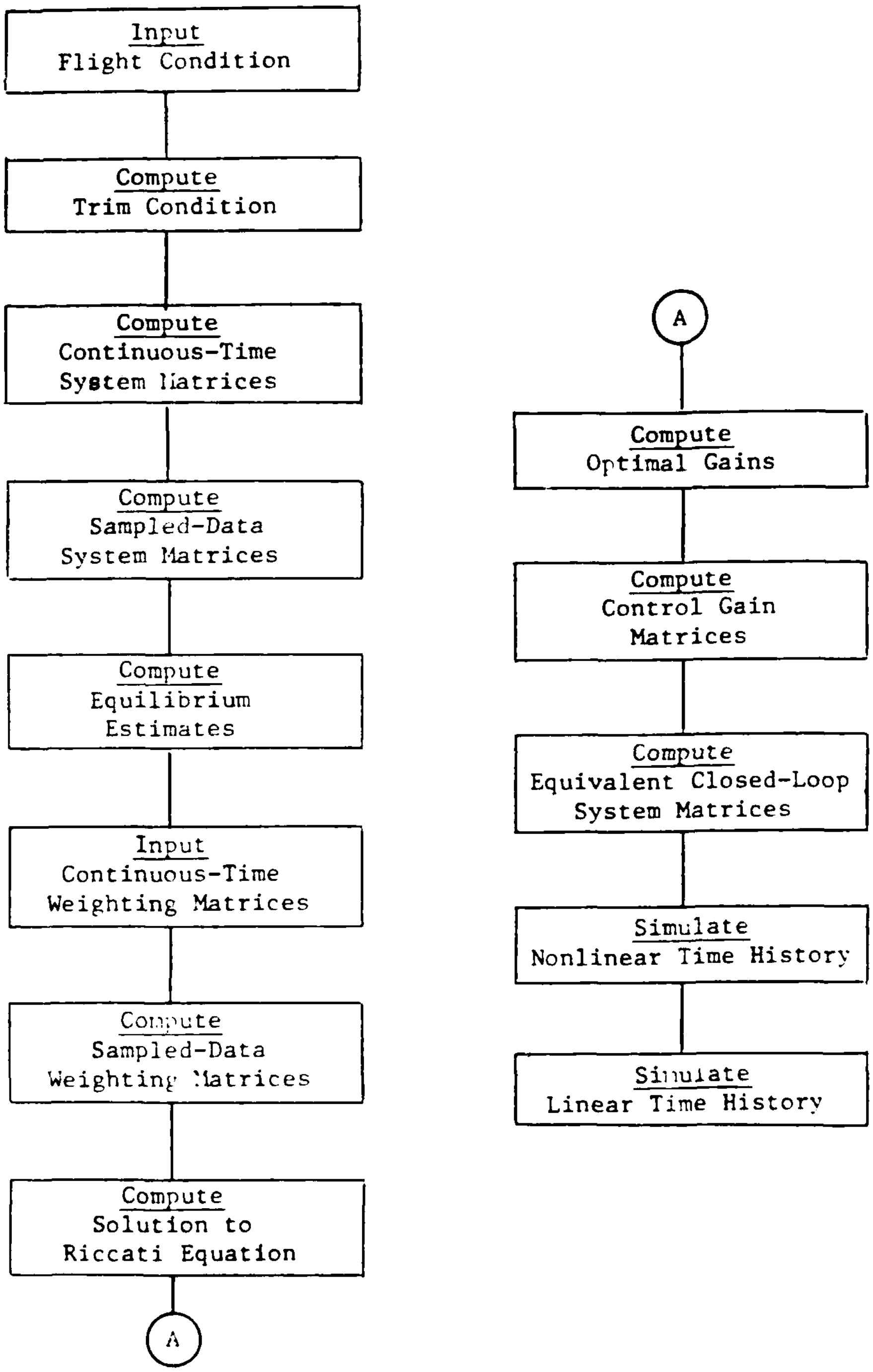


Figure 25: Gain Computation Flowchart

Table 14. Gain Computation Listings

```
FILE: CONTRL    FORTRAN  A                    PRINCETON UNIVERSITY TIME-SHARING SYSTEM


      DIMENSION WKF(4),PHI(4,4),F(4,4),FCOPY(4,4),QC(4,4),FINV(4,4),     CON00010
     1          G(4,2),GAMMA(4,2),HX(2,4),HU(2,2),RC(2,2),AREAP(3,3),     CON00020
     2          X0(4),U(2),TEMP1(4,4),TEMP2(2,4),TEMP3(2,2),TEMP4(4,2),  CON00030
     3          TEMP5(4,4),MHAT(4,2),MHATOT(4,2),QHAT(4,4),QHATOT(4,4),  CON00040
     4          Q(4,4),RHAT(2,2),RHATOT(2,2),R(2,2),QTNOW(4,4),           CON00050
     5          MTNOW(4,2),RTNOW(2,2),SINV(5,5),S(5,5),AREAS(4,4),        CON00060
     6          PHIT(4,4),GAMMAT(2,4),MHATT(2,4),PHITEM(4,4),             CON00070
     7          GAMTEM(4,2),PK(4,4),PKMNS1(4,4),APEAR(1,1),XT1(4),        CON00080
     8          C(2,4),S11(3,3),S12(3,2),S21(2,3),S22(2,2),C1(2,3),       CON00090
     9          C2(2,1),CP(2,2),CB(2,4),P1(3,3),P12(3,1),P2(1,1)          CON00100
      DIMENSION P21(1,3),G1(3,2),G2(1,2),TEMP6(2,1),CS(2,1),             CON00110
     1          TEMP7(1,2),TEMP8(1,2),CI(2,2),PHICL(4,4),PCL(4,4),        CON00120
     2          DY(2,1),DYT(2,1),X(4,1),U2(2,1),TEMP10(4,1),              CON00130
     3          TEMP11(2,1),PCLCPY(4,4),XOUT(4),U2OUT(2),TEMP12(1,1),     CON00140
     4          X1STAR(3,1),USTAR(2,1),XUSTAR(5,1),COMVEC(5,1),           CON00150
     5          X2STAR(1,1),TEMP9(3,1),PCHECK(4,4),IDEN(4,4),             CON00160
     6         KY(1,2),SXI(3,1),TEMP13(3,1),SXY(3,2),SUY(2,2),            CON00170
     7         HX1(2,3),HX2(2,1),SUI(2,1),FC(8,1),A1(7,9),A21(7,9),       CON00180
     8         CFEED1(7,1),CFEED2(7,1),FC1(9,1),FC2(18,1),A22(7,9),       CON00190
     9         T1(4,4),T2(4,4),T3(4,4),CTRIM(3,3),CVEC(3,1),AREACT(2,2) CON00200
      DIMENSION XI(4),UT(4),A2(7,18)                                     CON00210
      REAL IDEN                                                          CON00220
      COMPLEX WEIGEN(4),ZEIGEN(4,4)                                      CON00230
      REAL INTVL                                                         CON00240
      COMMON/CYCOFF/CY,CY0,CYB,CYDR,CYDA,CYP,CYR                         CON00250
      COMMON/CNCOEF/CN,CN0,CNB,CNDR,CNDA,CNP,CNR                         CON00260
      COMMON/CLCOFF/CL,CL0,CLB,CLDR,CLDA,CIP,CLR                         CON00270
      DATA A1/1.79,.7297,2.155,7.889,3.254,-3.595,.5348,-.06704,         CON00280
     1 -.02135,-.1052,-.3227,-.08524,.1556,-.006276,.8409E-3,.2326E-3,   CON00290
     2 .00148,.004372,.908E-3,-.002146,-.7038E-4,-.007062,-.02591,       CON00300
     3 -.01192,.005733,-.02486,.009345,-.1531,-.2718E-3,.8608E-3,        CON00310
     4 .5802E-3,.432E-3,-.001404,-.6244E-3,.004123,.5783E-5,-.1064E-4,   CON00320
     5 -.8374E-5,-.7653E-5,.274E-4,.9687E-5,-.4765E-4,.001574,.7078E-3,  CON00330
     6 .001884,-.001963,.001056,.0005536,.003235,-.2029E-4,-.3058E-4,    CON00340
     7 -.7971E-4,.3013E-4,.1207E-3,-.5157E-5,-.148E-3,.5743E-7,          CON00350
     8 .4931E-6,.1182E-5,.1268E-6,-.2462E-5,-.9082E-7,.2178E-5/          CON00360
      DATA A21/-1.936,-.05978,-.1887,-.9162,-.4223,-.658,-8.224,         CON00370
     1 -.2263,-.1526,-.6572,-3.385,.943,1.412,.6892,                     CON00380
     2 .06903,.001113,.005632,-.01312,.02269,.03657,.3341,               CON00390
     3 .1142,.000144,.03318,.155,-.04901,-.06353,.001572,-.6239E-3,      CON00400
     4 -.1153E-4,-.9239E-4,.3222E-3,-.333E-3,-.501E-3,-.004504,          CON00410
     5-.002133,-.1301E-3,-.4252E-3,-.002074,.7071E-3,.8178E-3,-.1901E-3,CON00420
     6 -.03905,.008944,.09175,.1234,-.06199,-.08771,.0363,               CON00430
     7 .1234,-.245E-3,-.03717,.3041,.1051,.0682,-.05388,                 CON00440
     8 .8762E-3,-.395E-3,-.001522,-.003296,.001981,.001415,-.002292/     CON00450
      DATA A22/.003238,-.7989E-4,-.001522,-.006569,-.003469,.8433E-4,    CON00460
     1 .001104,-.1553E-4,.6372E-5,.2116E-4,.4196E-4,-.2631E-4,-.2001E-4,CON00470
     2 .3346E-4,-.7741E-4,.1957E-5,.2149E-4,.4993E-4,.4469E-4,-.2753E-5,CON00480
     3-.8684E-5,.3322E-3,.4981E-3,-.00384,-.006807,.002308,.002882,      CON00490
     4.001139,-.00453,-.1815E-3,.00266,.001183,-.00517,-.003604,.002541,CON00500
     5 -.9593E-4,-.3247E-4,-.3302E-4,.1429E-3,-.4179E-4,.7385E-4,        CON00510
     6 .2666E-5,-.1424E-3,.5954E-5,.5423E-4,.3287E-4,.1639E-3,-.1119E-4, CON00520
     7 -.7E-4,.1853E-5,.3536E-6,.3215E-6,-.23E-5,.5309E-6,-.8737E-6,     CON00530
     8 -.7202E-6,.3768E-5,.498E-7,.2887E-7,.9122E-6,-.2211E-5,-.5247E-6,CON00540
     9 .1551E-5/                                                         CON00550
```

```
800    FORMAT('-','INPUT: FLIGHT CONDITION (A,TC,VEL,ALT)')
801    FORMAT('-','PHI EQUALS:',45X,'GAMMA EQUALS:')
802    FORMAT('0',4F10.3,13X,2F10.3)
811    FORMAT('-','S EQUALS:')
812    FORMAT('0',5F10.3)
829    FORMAT('-','STEADY STATE ESTIMATE')
860    FORMAT('-','OPEN LOOP? (1=YES,0=NO)')
803    FORMAT('-','INPUT: Q(SIDESLIP),Q(ROLL ANGLE) ')
804    FORMAT('-','INPUT: RC DIAGONAL ELEMENTS (2)')
805    FORMAT('-','Q(CONTINUOUS):',31X,'Q(DISCRETE):')
806    FORMAT('0',4F8.3,12X,4F8.3)
807    FORMAT('-','M(DISCRETE):')
808    FORMAT('0',2F8.3)
809    FORMAT('-','R(CONTINUOUS):',7X,'R(DISCRETE):')
810    FORMAT('0',2F8.3,4X,2F8.3)
813    FORMAT('-','P DID NOT CONVERGE IN ',I4,' STEPS')
814    FORMAT('-','P(STEADY STATE) EQUALS:')
815    FORMAT('0',4F10.3)
816    FORMAT('-','OPTIMAL GAINS (C) EQUAL:')
817    FORMAT('0',4F10.3)
818    FORMAT('-','CB EQUALS:',27X,'CF EQUALS:',11X,'CI EQUALS:')
819    FORMAT('0',4F8.3,5X,2F8.3,5X,2F8.3)
361    FORMAT('-','INPUT: SECONDS FOR TIME HISTORY')
820    FORMAT('-','DO YOU WISH TO RUN ANOTHER DELY VECTOR?',
      1         '(1=YES,0=NO)')
821    FORMAT('-','INPUT: DELY VECTOR (DEGREES) - (Y1,Y2)')
822    FORMAT('1','X(TRIM) EQUALS: (',4F10.3,') DEG')
823    FORMAT('0','U(TRIM) EQUALS: (',2F10.3,') DEG')
825    FORMAT(' ','TIME',4X,'YAW RATE',3X,'SIDESLIP',3X,'ROLL RATE',
      1         1X,'ROLL ANGLE',3X,'RUDDER',5X,'AILERON')
826    FORMAT(' ','----',4X,'--------',3X,'--------',3X,'---------',
      1         1X,'----------',3X,'------',5X,'-------')
850    FORMAT('0','ALL OUTPUT IN DEG OR DEG/SEC')
851    FORMAT('-','STATE TRANSITION MATRIX TIME HISTORY')
827    FORMAT(' ',F5.2,6(3X,F8.3))
830    FORMAT('-','DIFFERENT C MATRIX?(1=YES,0=NO)')
831    FORMAT('-','NONLINEAR TIME HISTORY? (1=YES,0=NO)')
832    FORMAT('-','LINEAR TIME HISTORY? (1=YES,0=NO)')
828    FORMAT('-','ANOTHER CASE?(1=YES,0=NO)')
870    FORMAT('-','FLIGHT CONDITION MATRIX EQUALS:')
871    FORMAT('0',8(F10.3,1X))
872    FORMAT('-','SCHEDULE GAINS? (1=YES,0=NO)')
873    FORMAT('-','SCHEDULED GAINS:')
C
C 100-----INITIALIZATION OF MODEL
C
C      INCLUDES: CREATION OF F,G,HX,AND HU MATRICES.
C      ALSO COMPUTATION OF SYSTEM EIGENVALUES
C
C
C INPUT FLIGHT CONDITION:  ANGLE OF ATTACK
C                          THROTTLE SETTING
C                          VELOCITY
C                          ALTITUDE
C
```

```
900    WRITE(6,800)
       READ(6,*)A,TC,VEL,ALT
C
C CREATE FLIGHT CONDITION MATRICES
C
       RHO=.002377*EXP(-ALT/25000.)
       DYNPRS=.5*RHO*VEL**2
       FC1(1,1)=EXP(-.001*TC)
       FC1(2,1)=FC1(1,1)*DYNPRS
       FC1(3,1)=FC1(2,1)*DYNPRS
       DO 10 I=1,6
10     FC1(I+3,1)=FC1(I,1)*A
       FC2(1,1)=1.
       FC2(2,1)=TC
       DO 15 I=1,4
15     FC2(I+2,1)=FC2(I,1)*DYNPRS
       DO 20 I=1,12
20     FC2(I+6,1)=FC2(I,1)*A
C
C INITIALIZE STATE AND CONTROL TO TRIM
C
       DO 100 I=1,4
100    X0(I)=0.
       DO 102  I=1,2
102    U(I)=0.
       CALL DNDYN(A,TC,VEL,ALT,X0,U,DX)
       CTRIM(1,1)=CYB
       CTRIM(1,2)=CYDR
       CTRIM(1,3)=CYDA
       CTRIM(2,1)=CNB
       CTRIM(2,2)=CNDR
       CTRIM(2,3)=CNDA
       CTRIM(3,1)=CLB
       CTRIM(3,2)=CLDR
       CTRIM(3,3)=CLDA
       CVEC(1,1)=CY0
       CVEC(2,1)=CN0
       CVEC(3,1)=CL0
       CNVRSN=180./3.14159
       CALL SMPLY(3,CTRIM,3,CNVRSN,CTRIM)
       CALL INVRS(3,CTRIM,2,APEACT,CTRIM)
       CALL SMPLY(3,CVEC,1,-1.,CVEC)
       CALL MMPLY(3,CTRIM,3,CVEC,1,CVEC)
       XT(1)=0.
       XT(2)=CVEC(1,1)
       XT(3)=0.
       XT(4)=0.
       UT(1)=CVEC(2,1)
       UT(2)=CVEC(3,1)
       DO 110 I=1,4
110    XOUT(I)=XT(I)*CNVRSN
       DO 112 I=1,2
112    U2OUT(I)=UT(I)*CNVRSN
       WRITE(6,822) (XOUT(I),I=1,4)
       WRITE(6,823) (U2OUT(I),I=1,2)
```

```
C
C COMPUTE F,G,HX,AND HU MATRICES AND SYSTEM EIGENVALUES
C
      CALL FANDG(A,TC,VEL,ALT,XT,UT,F,G,HX,HU,1)
      CALL EIGEN(F,FCOPY,4,0,WEIGEN,ZEIGEN,WKF)
C
C PARTITION F, G, AND HX FOR STEADY STATE RESPONSE AND COMPUTATION
C OF GAINS
C
C
C F
C
      DO 150 I=1,3
      DO 152 J=1,3
152   F1(I,J)=F(I,J)
150   CONTINUE
      DO 154 I=1,3
      F12(I,1)=F(I,4)
154   F21(1,I)=F(4,I)
      F2(1,1)=F(4,4)
C
C G
C
      DO 156 I=1,3
      DO 158 J=1,2
158   G1(I,J)=G(I,J)
156   CONTINUE
      DO 160 I=1,2
160   G2(1,I)=G(4,I)
C
C HX
C
      DO 115 I=1,2
      DO 117 J=1,3
117   HX1(I,J)=HX(I,J)
115   CONTINUE
      DO 114 I=1,2
114   HX2(I,1)=HX(I,4)
C
C COMPUTE STATE TRANSITION MATRIX AND CONTROL EFFECTS MATRIX
C
      CALL STM(4,F,.1,50,PHI,0)
      CALL CEM(4,F,2,G,.1,PHI,GAMMA,T1,T2,T3,IDEN)
C
C OUTPUT PHI AND GAMMA
C
      WRITE(6,801)
      DO 105 I=1,4
105   WRITE(6,802)(PHI(I,J),J=1,4),(GAMMA(I,K),K=1,2)
C
C 200-----COMPUTATION OF GAINS
C
C      INCLUDES: COMPUTATION OF OPTIMAL GAINS (C) AND THE GAIN
C      MATRICES CB, CF, AND CI. ALSO, THE STEADY STATE ESTIMATES
C      (FROM S MATRIX), COMPUTATION OF CONTINUOUS AND DISCRETE
```

```
C     WEIGHTING MATRICES, AND THE SOLUTION TO THE DISCRETE                   CON02210
C     RICCATI EQUATION (P).                                                  CON02220
C                                                                            CON02230
C                                                                            CON02240
C S MATRIX                                                                   CON02250
C                                                                            CON02260
C                                                                            CON02270
C FORM SINV                                                                  CON02280
C                                                                            CON02290
      DO 200 I=1,3                                                           CON02300
      DO 202 J=1,3                                                           CON02310
202   SINV(I,J)=F(I,J)                                                       CON02320
200   CONTINUE                                                               CON02330
      DO 204 I=1,3                                                           CON02340
      DO 206 J=1,2                                                           CON02350
      SINV(J+3,I)=HX(J,I)                                                    CON02360
206   SINV(I,J+3)=G(I,J)                                                     CON02370
204   CONTINUE                                                               CON02380
      DO 208 I=1,2                                                           CON02390
      DO 210 J=1,2                                                           CON02400
210   SINV(I+3,J+3)=HU(I,J)                                                  CON02410
208   CONTINUE                                                               CON02420
C                                                                            CON02430
C COMPUTE S                                                                  CON02440
C                                                                            CON02450
      CALL INVRS(5,SINV,4,AREAS,S)                                           CON02460
C                                                                            CON02470
C OUTPUT S                                                                   CON02480
C                                                                            CON02490
      WRITE(6,811)                                                           CON02500
      DO 213 I=1,5                                                           CON02510
213   WRITE(6,812)(S(I,J),J=1,5)                                             CON02520
C                                                                            CON02530
C PARTITION S FOR GAIN CALCULATIONS                                          CON02540
C                                                                            CON02550
      DO 290 I=1,3                                                           CON02560
      DO 291 J=1,3                                                           CON02570
291   S11(I,J)=S(I,J)                                                        CON02580
290   CONTINUE                                                               CON02590
      DO 292 I=1,3                                                           CON02600
      DO 293 J=1,2                                                           CON02610
      S12(I,J)=S(I,J+3)                                                      CON02620
293   S21(J,I)=S(J+3,I)                                                      CON02630
292   CONTINUE                                                               CON02640
      DO 294 I=1,2                                                           CON02650
      DO 295 J=1,2                                                           CON02660
295   S22(I,J)=S(I+3,J+3)                                                    CON02670
294   CONTINUE                                                               CON02680
C                                                                            CON02690
C KY                                                                         CON02700
C                                                                            CON02710
      CALL MMPLY(1,F21,3,S12,2,TEMP7)                                        CON02720
      CALL MMPLY(1,G2,2,S22,2,TEMP8)                                         CON02730
      CALL MADD(1,TEMP7,2,TEMP8,KY)                                          CON02740
C                                                                            CON02750
```

```
C SXI
C
      CALL MMPLY(3,S11,3,F12,1,SXI)
      CALL MMPLY(3,S12,2,HX2,1,TEMP13)
      CALL MADD(3,SXI,1,TEMP13,SXI)
      CALL SMPLY(3,SXI,1,-1.,SXI)
C
C SXY
C
      CALL MMPLY(3,S11,3,SXI,1,TEMP13)
      CALL MMPLY(3,TEMP13,1,KY,2,SXY)
      CALL MADD(3,S12,2,SXY,SXY)
C
C SUY
C
      CALL MMPLY(2,S21,3,SXI,1,TEMP6)
      CALL MMPLY(2,TEMP6,1,KY,2,SUY)
      CALL MADD(2,S22,2,SUY,SUY)
C
C SUI
C
      CALL MMPLY(2,S21,3,F12,1,SUI)
      CALL MMPLY(2,S22,2,HX2,1,TEMP6)
      CALL MADD(2,SUI,1,TEMP6,SUI)
      CALL SMPLY(2,SUI,1,-1.,SUI)
C
C AN ADDITIONAL STEADY STATE RESPONSE?
C
      GOTO 215
C
C INPUT DY VECTOR
C
215   DY(1,1)=10.
      DY(2,1)=0.
216   DO 220 I=1,2
220   DY(I,1)=DY(I,1)*3.14159/180.
C
C INITIALIZE XSTAR AND USTAR TO ZERO AT TIME=0
C
      DO 222 I=1,3
222   X1STAR(I,1)=0.
      DO 224 I=1,2
224   USTAR(I,1)=0.
      X2STAR(1,1)=0.
      TIME=0.
C
C OUTPUT HEADING
C
      WRITE(6,820)
      WRITE(6,850)
      WRITE(6,825)
      WRITE(6,826)
C
C ITERATE STEADY STATE RESPONSE FOR TWO SECONDS AT .2 SECOND INTERVALS
C
```

```
      DO 230 I=1,11
C
C X2STAR
C
      CALL SMPLY(1,KY,2,TIME,KYDT)
      CALL MMPLY(1,KYDT,2,DY,1,X2STAR)
C
C X1STAR
C
      CALL MMPLY(3,SXY,2,DY,1,X1STAR)
      CALL MMPLY(3,SXI,1,X2STAR,1,TEMP13)
      CALL MADD(3,X1STAR,1,TEMP13,X1STAR)
C
C USTAR
C
      CALL MMPLY(2,SUY,2,DY,1,USTAR)
      CALL MMPLY(2,SUI,1,X2STAR,1,TEMP6)
      CALL MADD(2,USTAR,1,TEMP6,USTAR)
C
C CONVERT XSTAR AND USTAR TO DEGREES FOR OUTPUT
C
      DO 236 J=1,3
236   XOUT(J)=X1STAR(J,1)*180./3.14159
      XOUT(4)=X2STAR(1,1)*180./3.14159
      DO 238 J=1,2
238   U2OUT(J)=USTAR(J,1)*180./3.14159
      WRITE(6,827)TIME,(XOUT(J),J=1,4),(U2OUT(K),K=1,2)
C
C STEP TIME .2 SECONDS
C
      TIME=.2*FLOAT(I)
230   CONTINUE
      IF(DY(1,1).EQ.0.) GOTO 240
      DY(1,1)=0.
      DY(2,1)=2.
      GOTO 215
C
C OPEN LOOP
C
240   WRITE(6,850)
      READ(6,*)IQUEST
      IF(IQUEST.EQ.1) GOTO 2403
C
C INITIALIZE STATE AND CONTROL WEIGHTING MATRICES
C
      DO 242 I=1,4
      DO 244 J=1,4
244   QC(1,J)=0.
242   CONTINUE
      DO 246 I=1,2
      DO 248 J=1,2
248   PC(I,J)=0.
246   CONTINUE
      QC(1,1)=1.
      QC(2,2)=10.
      QC(3,3)=1.
      QC(4,4)=25.
      RC(1,1)=1.
      RC(2,2)=.1
C
C COMPUTE THE DISCRETE WEIGHTING MATRICES USING SIMPSON'S RULE
C INTEGRATION.
C
C
C INITIALIZE TEMPORARY Q AND R
C
      DO 252 I=1,4
      DO 254 J=1,4
254   Q(I,J)=QC(I,J)
252   CONTINUE
      DO 256 I=1,2
      DO 258 J=1,2
258   R(I,J)=RC(I,J)
256   CONTINUE
C
C INITIALIZATION
C
      TNOW=0.
      INTVL=.1
      INDEX=0
      H=INTVL/10.
C
C QHAT,MHAT, AND RHAT AT T=0
C
      CALL STM(4,F,TNOW,50,PHITEM,0)
      CALL CEM(4,F,2,G,TNOW,PHITEM,GAMTEM,T1,T2,T3,IDEN)
      CALL TRNSPS(4,PHITEM,4,PHIT)
      CALL MMPLY(4,PHIT,4,Q,4,TEMP1)
      CALL MMPLY(4,TEMP1,4,PHITEM,4,QHATOT)
      CALL MMPLY(4,TEMP1,4,GAMTEM,2,MHATOT)
      CALL TRNSPS(4,GAMTEM,2,GAMMAT)
      CALL MMPLY(2,GAMMAT,4,Q,4,TEMP2)
      CALL MMPLY(2,TEMP2,4,GAMTEM,2,TEMP3)
      CALL MADD(2,R,2,TEMP3,RHATOT)
C
C ITERATE TO FIND QHAT,MHAT, AND RHAT FOR TIME INTERVAL = .1 SEC
C
      DO 260 I=1,10
      TNOW=TNOW+FLOAT(I)*H
      CALL STM(4,F,TNOW,50,PHITEM,0)
      CALL CEM(4,F,2,G,TNOW,PHITEM,GAMTEM,T1,T2,T3,IDEN)
      CALL TRNSPS(4,PHITEM,4,PHIT)
      CALL MMPLY(4,PHIT,4,Q,4,TEMP1)
      CALL MMPLY(4,TEMP1,4,PHITEM,4,QTNOW)
      CALL MMPLY(4,TEMP1,4,GAMTEM,2,MTNOW)
      CALL TRNSPS(4,GAMTEM,2,GAMMAT)
      CALL MMPLY(2,GAMMAT,4,Q,4,TEMP2)
      CALL MMPLY(2,TEMP2,4,GAMTEM,2,TEMP3)
      CALL MADD(2,R,2,TEMP3,RTNOW)
      IF(INDEX.EQ.0) GOTO 262
      TERM=2.                                                           CON04410
      INDEX=0                                                           CON04420
      GOTO 264                                                          CON04430
262   TERM=4.                                                           CON04440
      INDEX=1                                                           CON04450
264   IF(I.EQ.10) TERM=1.                                               CON04460
      CALL SMPLY(4,QTNOW,4,TERM,QTNOW)                                  CON04470
      CALL SMPLY(4,MTNOW,2,TERM,MTNOW)                                  CON04480
      CALL SMPLY(2,RTNOW,2,TERM,RTNOW)                                  CON04490
      CALL MADD(4,QHATOT,4,QTNOW,QHATOT)                                CON04500
      CALL MADD(4,MHATOT,2,MTNOW,MHATOT)                                CON04510
      CALL MADD(2,RHATOT,2,RTNOW,RHATOT)                                CON04520
260   CONTINUE                                                          CON04530
C                                                                       CON04540
C CALCULATE QHAT,MHAT, AND RHAT                                         CON04550
C                                                                       CON04560
      TERM=H/3.                                                         CON04570
      CALL SMPLY(4,QHATOT,4,TERM,QHAT)                                  CON04580
      CALL SMPLY(4,MHATOT,2,TERM,MHAT)                                  CON04590
      CALL SMPLY(2,RHATOT,2,TERM,RHAT)                                  CON04600
C                                                                       CON04610
C OUTPUT QC,RC,QHAT,MHAT,RHAT                                           CON04620
C                                                                       CON04630
      WRITE(6,805)                                                      CON04640
      DO 266 I=1,4                                                      CON04650
266   WRITE(6,806) (QC(I,J),J=1,4),(QHAT(I,K),K=1,4)                    CON04660
      WRITE(6,807)                                                      CON04670
      DO 268 I=1,4                                                      CON04680
268   WRITE(6,808) (MHAT(I,J),J=1,2)                                    CON04690
      WRITE(6,809)                                                      CON04700
      DO 269 I=1,2                                                      CON04710
269   WRITE(6,810) (RC(I,J),J=1,2),(RHAT(I,K),K=1,2)                    CON04720
C                                                                       CON04730
C SOLUTION OF THE DISCRETE RICCATI EQUATION                             CON04740
C                                                                       CON04750
      DO 270 I=1,4                                                      CON04760
      DO 272 J=1,4                                                      CON04770
      PK(I,J)=0.                                                        CON04780
272   PKMNS1(I,J)=0.                                                    CON04790
270   CONTINUE                                                          CON04800
      CALL TRNSPS(4,PHI,4,PHIT)                                         CON04810
      CALL TRNSPS(4,GAMMA,2,GAMMAT)                                     CON04820
      CALL TRNSPS(4,MHAT,2,MHATT)                                       CON04830
      PDET=1.                                                           CON04840
      INDCTR=0                                                          CON04850
      DO 275 I=1,1000                                                   CON04860
      CALL MMPLY(4,PK,4,PHI,4,TEMP1)                                    CON04870
      CALL MMPLY(4,PK,4,GAMMA,2,TEMP4)                                  CON04880
      CALL MMPLY(2,GAMMAT,4,TEMP1,4,TEMP2)                              CON04890
      CALL MMPLY(4,PHIT,4,TEMP1,4,TEMP1)                                CON04900
      CALL MMPLY(2,GAMMAT,4,TEMP4,2,TEMP3)                              CON04910
      CALL MADD(2,TEMP2,4,MHATT,TEMP2)                                  CON04920
      CALL MADD(2,TEMP3,2,RHAT,TEMP3)                                   CON04930
      CALL TRNSPS(2,TEMP2,4,TEMP4)                                      CON04940
      CALL INVPS(2,TEMP3,1,AREAR,TEMP3)                                 CON04950
      CALL MMPLY(4,TEMP4,2,TEMP3,2,TEMP4)
      CALL MMPLY(4,TEMP4,2,TEMP2,4,TEMP5)
      CALL SMPLY(4,TEMP5,4,-1.,TEMP5)
      CALL MADD(4,TEMP5,4,QHAT,TEMP5)
      CALL MADD(4,TEMP1,4,TEMP5,PKMNS1)
      PDET2=1.
      DO 276 J=1,4
      XT1(J)=0.
      DO 277 K=1,4
277   XT1(J)=XT1(J)+PKMNS1(J,K)
      IF(XT1(J).EQ.0.) GOTO 276
      PDET2=PDET2*XT1(J)
276   CONTINUE
      DO 278 J=1,4
      DO 279 K=1,4
279   PK(J,K)=PKMNS1(J,K)
278   CONTINUE
      DO 280 J=1,4
      DO 281 K=1,4
      IF(J.GE.K) GOTO 281
      PK(J,K)=.5*(PK(J,K)+PK(K,J))
      PK(K,J)=PK(J,K)
281   CONTINUE
280   CONTINUE
      DP=ABS((PDET2-PDET)/PDET)
      IF(DP.LT.(1.E-6)) GOTO 2401
      PDET=PDET2
275   CONTINUE
C
C ERROR MESSAGE FOR NONCONVERGENCE
C
      WRITE(6,813)I
C
C OUTPUT P(STEADY STATE)
C
2401  WRITE(6,814)
      DO 282 I=1,4
282   WRITE(6,815)(PK(I,J),J=1,4)
C
C COMPUTATION OF OPTIMAL GAINS, C
C
      CALL MMPLY(4,PK,4,GAMMA,2,TEMP4)
      CALL MMPLY(4,PK,4,PHI,4,TEMP1)
      CALL MMPLY(2,GAMMAT,4,TEMP4,2,TEMP3)
      CALL MMPLY(2,GAMMAT,4,TEMP1,4,TEMP2)
      CALL MADD(2,TEMP3,2,RHAT,TEMP3)
      CALL MADD(2,TEMP2,4,MHATT,TEMP2)
      CALL INVRS(2,TEMP3,1,AREAL,TEMP3)
      CALL MMPLY(2,TEMP3,2,TEMP2,4,C)
      GOTO 2402
C
C IF OPEN LOOP, OPTIMAL GAINS ARE ZERO
C
2403  DO 285 I=1,2
      DO 286 J=1,4
```

```
286    C(I,J)=0.
285    CONTINUE
C
C OUTPUT OPTIMAL GAINS
C
2402   WRITE(6,816)
       DO 287 I=1,2
287    WRITE(6,817) (C(I,J),J=1,4)
C
C COMPUTATION OF CB, CF, AND CI
C
C
C PARTITION F,G,H,C
C
       DO 297 I=1,2
       DO 298 J=1,3
298    C1(I,J)=C(I,J)
297    CONTINUE
       DO 299 I=1,2
299    C2(I,1)=C(I,4)
C
C COMPUTE GAIN MATRICES - CB,CF,CI
C
C
C CB
C
       CALL SMPLY(2,C,4,-1.,CB)
C
C CF
C
       CALL MMPLY(2,C1,3,SXY,2,CF)
       CALL MADD(2,CF,2,SUY,CF)
C
C CS
C
       CALL MMPLY(2,C1,3,SXI,1,CS)
       CALL MADD(2,CS,1,C2,CS)
       CALL MADD(2,SUI,1,CS,CS)
C
C CI
C
       CALL MMPLY(2,CS,1,KY,2,CI)
C
C OUTPUT GAIN MATRICES
C
       WRITE(6,818)
       DO 2100 I=1,2
2100   WRITE(6,819) (CB(I,J),J=1,4),(CF(I,K),K=1,2),(CI(I,L),L=1,2)
C
C SCHEDULED GAINS?
C
       WRITE(6,972)
       READ(5,*)IQUEST
       IF(IQUEST.NE.1) GOTO 300
       DO 2225 I=1,7
```

```
      DO 2230 J=1,9
      A2(I,J)=A21(I,J)
2230  A2(I,J+9)=A22(I,J)
2225  CONTINUE
875   FORMAT('-','FC1 EQUALS: ',9(E10.4,1X))
876   FORMAT('-','A1  EQUALS: ',9(E10.4,1X))
877   FORMAT(' ','             ',9(E10.4,1X))
      WRITE(6,875) (FC1(I,1),I=1,9)
      WRITE(6,876) (A1(1,I),I=1,9)
      DO 2200 I=1,6
2200  WRITE(6,877) (A1(I+1,J),J=1,9)
878   FORMAT('-','FC2 EQUALS: ',9(E10.4,1X))
879   FORMAT('-','A2  EQUALS: ',9(E10.4,1X))
880   FORMAT(' ','             ',9(E10.4,1X))
      WRITE(6,878) (FC2(I,1),I=1,9)
      WRITE(6,879) (A2(1,I),I=1,9)
      DO 2201 I=1,6
2201  WRITE(6,880) (A2(I+1,J),J=1,9)
      WRITE(6,878) (FC2(I,1),I=10,18)
      WRITE(6,879) (A2(1,I),I=10,18)
      DO 2202 I=1,6
2202  WRITE(6,880) (A2(I+1,J),J=10,18)
      CALL MMPLY(7,A1,9,FC1,1,CFEED1)
      CALL MMPLY(7,A2,18,FC2,1,CFEED2)
      CB(1,1)=CFEED1(1,1)
      CB(2,1)=CFEED1(2,1)
      CB(2,3)=CFEED1(3,1)
      CB(2,4)=CFEED1(4,1)
      CF(1,2)=CFEED1(5,1)
      CF(2,1)=CFEED1(6,1)
      CF(2,2)=CFEED1(7,1)
      CB(1,2)=CFEED2(1,1)
      CB(1,3)=CFEED2(2,1)
      CB(1,4)=CFEED2(3,1)
      CB(2,2)=CFEED2(4,1)
      CF(1,1)=CFEED2(5,1)
      CI(1,1)=CFEED2(6,1)
      CI(2,1)=CFEED2(7,1)
      CI(1,2)=0.
      CI(2,2)=0.
      WRITE(6,873)
      WRITE(6,913)
      DO 2220 I=1,2
2220  WRITE(6,913) (CB(I,J),J=1,4),(CF(I,K),K=1,2),(CI(I,L),L=1,2)
C
C 300-----COMPUTATION OF CLOSED LOOP F
C
C     INCLUDES: COMPUTATION OF FCL AND ITS EIGENVALUES
C
300   CALL MMPLY(4,GAMMA,2,CB,4,TEMP1)
      CALL MADD(4,PHI,4,TEMP1,PHICL)
      CALL CLOOPF(4,PHICL,.1,FCL,1)
      CALL EIGEN(FCL,FCLCPY,4,0,WEIGEN,ZEIGEN,WKF)
C
C 400-----TIME HISTORY
```

```
C
C
C NONLINEAR TIME HISTORY (4TH ORDER RUNGA-KUTTA INTEGRATION)
C
      WRITE(6,831)
      READ(6,*)IQUEST
      IF(IQUEST.NE.1) GOTO 460
      CALL RKINT(A,TC,VEL,ALT,CB,CF,CI,XT,UT)
C
C LINEAR TIME HISTORY (STATE TRANSITION MATRIX)
C
460    WRITE(6,832)
      READ(6,*)IQUEST
      IF(IQUEST.NE.1) GOTO 500
      IF(IQUEST.NE.1) GOTO 500
      INDCTR=0
      INTVL=.1
      ISEC=5
      ITER=10*ISEC
      IOUT=ITER/50
      GOTO 405
405   DY(1,1)=10.
      DY(2,1)=0.
402   DO 406  I=1,2
406   DY(I,1)=DY(I,1)*3.14159/180.
      DO 410 I=1,4
410   X(I,1)=0.
      DO 411 I=1,4
411   XOUT(I)=X(I,1)*180./3.14159
      WRITE(6,851)
      WRITE(6,852)
      WRITE(6,825)
      WRITE(6,826)
      CALL MMPLY(2,CF,2,DY,1,U2)
      DO 440 I=1,2
440   U2OUT(I)=U2(I,1)*180./3.14159
      TIME=0.
      WRITE(6,827)TIME,(XOUT(I),I=1,4),(U2OUT(J),J=1,2)
      DO 415 I=1,ITER
      INDCTR=INDCTR+1
      TIME=INTVL*FLOAT(I)
      CALL MMPLY(4,GAMMA,2,U2,1,TEMP10)
      CALL MMPLY(4,PHI,4,X,1,X)
      CALL MADD(4,X,1,TEMP10,X)
      DYT(1,1)=DY(1,1)*TIME
      DYT(2,1)=DY(2,1)*TIME
      CALL MMPLY(2,CI,2,DYT,1,TEMP6)
      CALL MMPLY(2,CF,2,DY,1,TEMP11)
      CALL MADD(2,TEMP11,1,TEMP6,TEMP6)
      CALL MMPLY(2,C3,4,X,1,TEMP11)
      CALL MADD(2,TEMP6,1,TEMP11,U2)
      IF(INDCTR.NE.IOUT) GOTO 415
      DO 421 J=1,4
421   XOUT(J)=X(J,1)*180./3.14159
      DO 420 J=1,2
```

```
426   U2OUT(J)=U2(J,1)*180./3.14159
      WRITE(6,827)TIME,(XOUT(K),K=1,4),(U2OUT(J),J=1,2)
      INDCTR=0
415   CONTINUE
      IF(DY(1,1).EQ.0.) GOTO 500
      DY(1,1)=0.
      DY(2,1)=2.
      GOTO 402
500   WRITE(6,830)
      READ(6,*)IQUEST
      IF(IQUEST.EQ.1) GOTO 240
      WRITE(6,829)
      READ(6,*)IQUEST
      IF(IQUEST.EQ.1) GOTO 900
      STOP
      END
```

```
C
C THE FOLLOWING SUBROUTINE COMPUTES THE NONLINEAR EQUATIONS OF
C MOTION BY ACCEPTING THE CURRENT STATE (X) AND THE FLIGHT
  CONDITION (A,TC,VEL) AND RETURNING THE STATE RATES (DX)
C
      SUBROUTINE DNDYN(A,TC,VEL,ALT,X,U,DX)
      DIMENSION X(4),DX(4),U(2)
      COMMON/CNTRLS/DR,DA,BETA
      COMMON/CYCOEF/CY,CY0,CYB,CYDR,CYDA,CYP,CYR
      COMMON/CNCOEF/CN,CN0,CNB,CNDR,CNDA,CNP,CNR
      COMMON/CLCOEF/CL,CL0,CLB,CLDR,CLDA,CLP,CLR
      COMMON/CYCOMP/CY0T,CY00,CYBT,CYB0,CYDRT,CYDR0
      COMMON/CNCOMP/CN0T,CN00,CNBT,CNB0,CNDRT,CNDR0
      COMMON/CLCOMP/CL0T,CL00,CLBT,CLB0,CLDRT,CLDR0
      COMMON/PCOMP/CLPGAM,CLACLA,CLPDGT,CLPDG0,K,CNPCL
      COMMON/RCOMP/CLRCL,CLIFTT,CLIFT0,CDRAGT,CDRAG0,CYPCL
      COMMON/CNSTNT/MACH
      REAL MACH,IX,IZ,K,MASS
C
C COMPUTE CONSTANTS REQUIRED FOR COMPUTATIONS:
C     SOS - SPEED OF SOUND (FEET/SEC)
C     ALT - ALTITUDE (FEET)
C     RHO - DENSITY (SLUGS/FEET**3)
C     IX - MOMENT OF INERTIA ABOUT X (SLUGS*FEET**2)
C     IZ - MOMENT OF INERTIA ABOUT Z (SLUGS*FEET**2)
C     B - SPAN (FEET)
C     DYNPRS - DYNAMIC PRESSURE (LBS/(FEET**2))
C     MACH - LOCAL MACH NUMBER
C     S - WING AREA (FEET**2)
C     MASS - AIRCRAFT MASS (LBS*SEC**2/FEET)
C     GRAV - GRAVITATIONAL CONSTANT (FEET/(SEC**2)
C
      FPA=0.
      PI = 3.14159
      BETA = X(2)*180/PI
      ARAD = A*PI/180.
      RHO = .002377*EXP(-ALT/25000.)
      SOS = 1116.-(.00409)*(ALT)
      MACH = VEL/SOS
      B = 33.38
      IX = 1284.78
      IZ = 3234.72
      S = 184.
      DYNPRS = .5*RHO*(VEL**2)
      GRAV = 32.174
      MASS = 2948./GRAV
      DR=U(1)*180./3.14159
      DA=U(2)*180./3.14159
      ZP=2.87
      LP=17.29
      Z=ZP*COS(ARAD)-LP*SIN(ARAD)
C
C INITIALIZE LONGITUDINAL STATE VARIABLES (Q,U,W)
C
      Q = 0
      VEL1 = (VEL**2-(SIN(X(2))*VEL)**2)**.5
      UVEL = VEL1
      W = 0.
C
C THE FOLLOWING PORTION COMPUTES THE STABILITY AND CONTROL
C DERIVITIVES GIVEN THE CURRENT FLIGHT CONDITION.
C
C
C CONSTANT COMPONENTS
C
      CYOT = ((((((1.094E-8*A-7.104E-7)*A+1.63E-5)*A-1.496E-4)*A
     1      +2.979E-4)*A+2.448E-3)*A-1.715E-2)*A
      CYOO = ((((((-1.996E-9*A+1.298E-7)*A-2.974E-6)*A+2.713E-5)*A
     1      -5.525E-5)*A-4.614E-4)*A+1.812E-3)*A
      CYO = CYOT*TC+CYOO
      CNOT = ((((((5.553E-9*A-3.259E-7)*A+6.695E-6)*A-5.416E-5)*A
     1      +7.633E-5)*A+1.025E-3)*A-4.504E-3)*A-.03281
      CNOO = ((((((3.817E-10*A-2.568E-8)*A+6.104E-7)*A-5.881E-6)*A
     1      +1.52E-5)*A+8.592E-5)*A-5.593E-4)*A+.001822
      CNO = CNOT*TC+CNOO
      CLO = ((((((-7.099E-10*A+4.458E-8)*A-9.659E-7)*A+7.877E-6)*A
     1      -6.971E-6)*A-1.637E-4)*A+2.167E-4)*A+.00717
C
C SIDESLIP (B) DERIVATIVES
C
      CYBT = ((((((1.274E-9*A-8.63E-8)*A+2.089E-6)*A-2.044E-5)*A
     1      +4.791E-5)*A+3.116E-4)*A-1.607E-3)*A-.0188
      CYBO = .000238*A-.01249
      CYB = CYBT*TC+CYBO
      CNBT = ((((((-2.604E-11*A+2.584E-9)*A-8.596E-8)*A+1.123E-6)*A
     1      -4.376E-6)*A-2.051E-5)*A+3.196E-4)*A+.001
      CNBO = ((((((3.633E-12*A-4.747E-10)*A+1.815E-8)*A-2.599E-7)*A
     1      +1.068E-6)*A+3.967E-6)*A-8.049E-5)*A+.002
      CNB = CNBT*TC+CNBO
      CLBT = ((((((-3.283E-10*A+2.31E-8)*A-5.742E-7)*A+5.638E-6)*A
     1      -1.109E-5)*A-9.937E-5)*A+4.282E-4)*A-.00106
      CLBO = ((((((6.597E-11*A-4.466E-9)*A+1.063E-7)*A-1.012E-6)*A
     1      +1.852E-6)*A+1.433E-5)*A-1.463E-5)*A-.00155
      CLB = CLBT*TC+CLBO
C
C RUDDER (DR) DERIVATIVES
C
      CYDRT = ((((((6.225E-10*A-4.079E-8)*A+9.409E-7)*A-8.651E-6)*A
     1      +1.617E-5)*A+1.582E-4)*A-2.147E-4)*A+.0068
      CYDRO = ((((((4.637E-11*A-3.079E-9)*A+7.235E-8)*A-6.912E-7)*A
     1      +1.695E-6)*A+6.535E-6)*A-1.208E-5)*A+.00296
      CYDR = CYDRT*TC+CYDRO
      CNDRT = ((((((-3.538E-11*A+4.476E-9)*A-7.65E-8)*A+5.046E-7)*A
     1      -1.104E-6)*A-4.531E-6)*A-8.116E-5)*A-.0043
      CNDRO = ((((((-7.266E-11*A+4.832E-9)*A-1.141E-7)*A+1.077E-6)*A
     1      -2.106E-6)*A-1.69E-5)*A+2.946E-5)*A-.00156
      CNDR = CNDRT*TC+CNDRO
      CLDRT = ((((((3.278E-10*A-2.428E-8)*A+6.533E-7)*A-7.27E-6)*A
     1      +2.069E-5)*A+1.294E-4)*A-6.396E-4)*A+.00168
      CLDRO = ((((((-1.381E-10*A+9.568E-9)*A-2.394E-7)*A+2.487E-6)*A
     1     -6.88E-6)*A-4.086E-5)*A+1.717E-4)*A+.000247
      CLDR = CLDRT*TC+CLDR0

 AILERON (DA) DERIVITIVES

      CYDA = ((((((2.433E-11*A-1.603E-9)*A+3.859E-8)*A-3.817E-7)*A
     1     +8.667E-7)*A+5.518E-6)*A-2.085E-5)*A-.000267
      CNDA = ((((((-1.637E-11*A+1.088E-9)*A-2.596E-8)*A+2.497E-7)*A
     1     -5.133E-7)*A-4.023E-6)*A+3.243E-5)*A-.000005
      CLDA = ((((((-1.381E-11*A+9.8E-10)*A-2.405E-8)*A+2.227E-7)*A
     1     -2.285E-7)*A-5.377E-6)*A+3.619E-6)*A-.00127

 COMPUTE CLIFT AND CDRAG TO HELP CALCULATE P AND R DERIVATIVES

      CLIFTT = ((((((2.78E-8*A-1.81E-6)*A+4.299E-5)*A-4.463E-4)*A
     1     +1.536E-3)*A+6.761E-3)*A-1.321E-3)*A+.357
      CLIFT0 = ((((((-5.571E-10*A+8.952E-8)*A-3.684E-6)*A+5.42E-5)*A
     1     -2.531E-4)*A-9.214E-4)*A+.08337)*A+.121
      CLIFT = CLIFTT*TC+CLIFT0
      CDRAGT = ((((((-1.401E-8*A+1.045E-6)*A-2.756E-5)*A+2.968E-4)*A
     1     -9.036E-4)*A-.003391)*A+.02157)*A-1.053
      CDRAG0 = ((((((-6.539E-10*A+2.608E-8)*A-1.176E-7)*A-5.244E-6)*A
     1     +7.751E-5)*A-2.278E-5)*A+4.263E-5)*A+.0551
      CDRAG = CDRAGT*TC+CDRAG0

  OLL RATE (P) DERIVITIVES

      CLPGAM = (((-4.012*MACH+7.273)*MACH-4.379)*MACH+.8069)*MACH
     1     -.426
      CLACLA = ((((((-4.844E-10*A+1.797E-7)*A-6.524E-6)*A+6.049E-5)*A
     1     -2.235E-5)*A-1.326E-3)*A-1.72E-3)*A+.99
      CLPDGT = ((((((8.453E-10*A-5.086E-8)*A+1.066E-6)*A-9.049E-6)*A
     1     +1.985E-5)*A+2.109E-4)*A+9.369E-5)*A+.1329
      CLPDG0 = ((((((2.325E-11*A-5.324E-10)*A-1.689E-8)*A+4.544E-7)*A
     1     -3.006E-6)*A+5.38E-5)*A+2.386E-4)*A-.00675
      CLPDRG = CLPDGT*TC+CLPDG0
      CLPW = (-.42)*(.936)*CLACLA*.985+CLPDRG
      TERM=2*(Z/B)*(Z-ZP)/B
      IF(TERM.LT.0.) TERM=-TERM
      CLP=CLPW+.5*(-.314)*.234*.155+TERM*(-.181)
      K=((((((4.722E-8*A-1.83E-6)*A+4.235E-6)*A+.0005208)*A-
     1        .0059)*A+.00723)*A+.087)*A+1.00
      CYPW = K*(-.06*CL)+.161
      CYP=CYPW+2*((Z-ZP)/B)*(-.181)
      CNPW = -CLP*TAN(ARAD)-K*(-CLP*TAN(ARAD)+.1003*CLIFT)
      CNP=CNPW-(2/3)*(LP*COS(ARAD)+ZP*SIN(ARAD))*((Z-ZP)/B)*(-.181)

      RATE (R) DERIVITIVES

      CLRW = CLIFT*.241+.001079
      CLR = CLRW-(2/(B**2))*(LP*COS(ARAD)+ZP*SIN(ARAD))*Z*(-.181)
      CD0 = CDRAG0-(CLIFT**2)/(6.04*PI)
      CNRW = -.02*CLIFT**2-.32*CD0
      CNR = CNRW+(2/(B**2))*((LP*COS(ARAD)+ZP*SIN(ARAD))**2)*(-.181)
      CYR = 0
```

```
C                                                                       LDD01660
C COMPUTE COEFFICIENTS                                                   LDD01670
C                                                                       LDD01680
      CY  = CY0+CYB*BETA+CYDR*DR+CYDA*DA+(CYP*X(3)+CYR*X(1))*B/(2*VEL)   LDD01690
      CN  = CN0+CNB*BETA+CNDR*DR+CNDA*DA+(CNP*X(3)+CNR*X(1))*B/(2*VEL)   LDD01700
      CL  = CL0+CLB*BETA+CLDR*DR+CLDA*DA+(CLP*X(3)+CLR*X(1))*B/(2*VEL)   LDD01710
C                                                                       LDD01720
C COMPUTE STATE RATES                                                   LDD01730
C                                                                       LDD01740
C     THE STATE RATES ARE CALCULATED IN THE FOLLOWING ORDER:            LDD01750
C                                                                       LDD01760
C        DX(1)  -  RDOT  (RAD/SEC**2)                                   LDD01770
C        DX(2)  -  BDOT  (RAD/SEC)                                      LDD01780
C        DX(3)  -  PDOT  (RAD/SEC**2)                                   LDD01790
C        DX(4)  -  PHIDOT  (RAD/SEC)                                    LDD01800
C                                                                       LDD01810
      DX(1)  =  (DYNPRS*S*B*CN)/IZ                                      LDD01820
      DX(2)  =  (-X(1)*UVEL+X(3)*W+GRAV*COS(FPA)*SIN(X(4))+             LDD01830
     1        (DYNPRS*S*CY)/(MASS))/VEL                                 LDD01840
      DX(3)  =  (DYNPRS*S*B*CL)/IX                                      LDD01850
      DX(4)  =  X(3)+(Q*SIN(X(4))+X(1)*COS(X(4)))*TAN(FPA)              LDD01860
C                                                                       LDD01870
C                                                                       LDD01880
C  END OF NONLINEAR EQUATION SUBROUTINE                                 LDD01890
C                                                                       LDD01900
C                                                                       LDD01910
      RETURN                                                            LDD01920
      END                                                               LDD01930
```

```
C THIS PROGRAM GENERATES F AND G MATRICES FOR DIFFERENT FLIGHT CONDITION
C THE FLIGHT CONDITION IS SPECIFIED BY ANGLE OF ATTACK, A, THROTTLE
C SETTING, TC, AND VELOCITY, VEL.
C
      SUBROUTINE FANDG(A,TC,VEL,ALT,X,U,F,G,HX,HU,IOUT)
      DIMENSION F(4,4),G(4,2),X(4),XT(4),DX(4),DX1(4),HX(2,4),YPLUS(2),
     1              YMINUS(2),U(2),HU(2,2),UT(2)
      REAL NR
      COMMON/CNTRLS/DR,DA
      COMMON/CYCOEF/CY,CY0,CYB,CYDR,CYDA,CYP,CYR
      COMMON/CNCOEF/CN,CN0,CNB,CNDR,CNDA,CNP,CNR
      COMMON/CLCOEF/CL,CL0,CLB,CLDR,CLDA,CLP,CLR
100   FORMAT('0','THE FOLLOWING F,G AND H MATRICES WERE GENERATED')
101   FORMAT(' ','UNDER THE FOLLOWING FLIGHT CONDITION:')
102   FORMAT('-',10X,'ANGLE OF ATTACK = ',F5.2,' DEG')
103   FORMAT(' ',10X,'THROTTLE SETTING = ',F6.4)
104   FORMAT(' ',10X,'VELOCITY = ',F8.4,' FEET/SEC')
105   FORMAT(' ',10X,'ALTITUDE = ',F10.3,' FEET')
106   FORMAT('-','F EQUALS:',47X,'G EQUALS:')
107   FORMAT('0',4F10.3,13X,2F10.3)
108   FORMAT('-','HX EQUALS:',46X,'HU EQUALS:')
109   FORMAT('0','DET(F) EQUALS:',F10.3)
110   FORMAT('-','   CY0          CYB         CYDR        CYDA          ',
     1          'CYP          CYR')
111   FORMAT('-','   CN0          CNB         CNDR        CNDA          ',
     1          'CNP          CNR')
112   FORMAT('-','   CL0          CLB         CLDR        CLDA          ',
     1          'CLP          CLR')
113   FORMAT('0',6(E10.3,2X))
C
C COMPUTE F USING PERTURBATION METHOD
C
      DELX = .1
      DO 150 I50=1,4
      DO 151 I51=1,4
151   XT(I51) = X(I51)
      XT(I50) = X(I50) + .5 * DELX
      CALL DNDYN(A,TC,VEL,ALT,XT,U,DX)
      XT(I50) = X(I50) - .5 * DELX
      CALL DNDYN(A,TC,VEL,ALT,XT,U,DX1)
      DO 152 I52=1,4
152   F(I52,I50) = (DX(I52) - DX1(I52))/DELX
150   CONTINUE
C
C COMPUTE G USING PERTURBATION METHOD
C
      DELU=.1
      DO 153 I53=1,2
      DO 154 I54=1,2
154   UT(I54)=U(I54)
      UT(I53)=U(I53)+.5*DELU
      CALL DNDYN(A,TC,VEL,ALT,X,UT,DX)
      UT(I53)=U(I53)-.5*DELU
      CALL DNDYN(A,TC,VEL,ALT,X,UT,DX1)
      DO 1551 I55=1,4
```

```
1551  G(I55,I53)=(DX(I55)-DX1(I55))/DELU                                    FAN00560
153   CONTINUE                                                              FAN00570
C                                                                           FAN00580
C COMPUTE HX USING PERTURBATION METHOD                                       FAN00590
C                                                                           FAN00600
      DO 155 I55=1,4                                                        FAN00610
      DO 156 I56=1,4                                                        FAN00620
156   XT(I56)=X(I56)                                                        FAN00630
      XT(I55)=X(I55)+.5*DELX                                                FAN00640
      YPLUS(1)=XT(3)                                                        FAN00650
      YPLUS(2)=XT(2)                                                        FAN00660
      XT(I55)=X(I55)-.5*DELX                                                FAN00670
      YMINUS(1)=XT(3)                                                       FAN00680
      YMINUS(2)=XT(2)                                                       FAN00690
      DO 157 I57=1,2                                                        FAN00700
157   HX(I57,I55)=(YPLUS(I57)-YMINUS(I57))/DELX                             FAN00710
155   CONTINUE                                                              FAN00720
C                                                                           FAN00730
C COMPUTE HU USING PERTURBATION METHOD                                       FAN00740
C                                                                           FAN00750
      DO 158 I58=1,2                                                        FAN00760
      DO 159 I59=1,2                                                        FAN00770
159   UT(I59)=U(I59)                                                        FAN00780
      UT(I58)=U(I58)+.5*DELU                                                FAN00790
      YPLUS(1)=X(3)                                                         FAN00800
      YPLUS(2)=X(2)                                                         FAN00810
      UT(I58)=U(I58)-.5*DELU                                                FAN00820
      YMINUS(1)=X(3)                                                        FAN00830
      YMINUS(2)=X(2)                                                        FAN00840
      DO 160 I60=1,2                                                        FAN00850
160   HU(I60,I58)=(YPLUS(I60)-YMINUS(I60))/.05                              FAN00860
158   CONTINUE                                                              FAN00870
C                                                                           FAN00880
C CALCULATE DETERMINANT OF F                                                 FAN00890
C                                                                           FAN00900
      CALL DTRMNT(4,F,DETF)                                                 FAN00910
C                                                                           FAN00920
C OUTPUT FLIGHT CONDITION AND F AND G MATRICES                               FAN00930
C                                                                           FAN00940
      IF(IOUT.NE.1) GOTO 600                                                FAN00950
      WRITE(6,100)                                                          FAN00960
      WRITE(6,101)                                                          FAN00970
      WRITE(6,102)A                                                         FAN00980
      WRITE(6,103)TC                                                        FAN00990
      WRITE(6,104)VEL                                                       FAN01000
      WRITE(6,105)ALT                                                       FAN01010
      WRITE(6,106)                                                          FAN01020
      DO 161 I61=1,4                                                        FAN01030
161   WRITE(6,107)(F(I61,I62),I62=1,4),(G(I61,I63),I63=1,2)                 FAN01040
      WRITE(6,109)DETF                                                      FAN01050
      WRITE(6,108)                                                          FAN01060
      DO 164 I64=1,2                                                        FAN01070
164   WRITE(6,107)(HX(I64,I65),I65=1,4),(HU(I64,I66),I66=1,2)               FAN01080
600   RETURN                                                                FAN01090
      END                                                                   FAN01100
```

```
C
C THIS SUBROUTINE COMPUTES THE STATE TRANSITION MATRIX FOR A SYSTEM
C SPECIFIED BY THE MATRIX A AND THE TIME STEP T. IT ITERATES TO
C FIND A SOLUTION WHICH CONVERGES TO WITHIN .1% AFTER KEND ITERATIONS.
C THE STATE TRANSITION MATRIX IS THEN PLACED IN PHI.
C
      SUBROUTINE STM(ORDER,A,T,KEND,PHI,OUTPUT)
      INTEGER ORDER,OUTPUT
      DIMENSION A(ORDER,ORDER),A1(12,12),A2(12,12),
     1          A3(12,12),IDEN(4,4),PHI(ORDER,ORDER),
     2          PHI1(12,12),TEMP(12,12),T1(4,4),T2(4,4),T3(4,4)
      REAL IDEN
100   FORMAT('-','***WARNING-THE STATE TRANSITION MATRIX DID NOT')
101   FORMAT(' ','CONVERGE IN THE ',I2,'TH ORDER***')
102   FORMAT('-','THE STATE TRANSITION MATRIX CONVERGED IN ',I2,
     1       'TH ORDER')
103   FORMAT('-','THE STATE TRANSITION MATRIX IS AS FOLLOWS:')
104   FORMAT('0',10(2X,F8.3))
C
C INITIALIZE IDENTITY MATRIX TO THE ORDER OF THE SYSTEM
C
      DO 200 I=1,ORDER
      DO 201 J=1,ORDER
      IDEN(I,J)=0.
201   CONTINUE
200   CONTINUE
      DO 202 I=1,ORDER
      IDEN(I,I)=1.
202   CONTINUE
C
C INITIALIZE:      PHI = I
C                  A1 = A*T
C                  A3 = I
C
      DO 203 I=1,ORDER
      DO 204 J=1,ORDER
      PHI(I,J)=IDEN(I,J)
      A3(I,J)=IDEN(I,J)
      A1(I,J)=A(I,J)*T
204   CONTINUE
203   CONTINUE
C
C START LOOP TO CONVERGE TO STATE TRANSITION MATRIX
C
      DO 205 K=1,KEND
C
C A2 = A*T/K
C
      DO 206 I=1,ORDER
      DO 207 J=1,ORDER
      A2(I,J)=A1(I,J)/FLOAT(K)
207   CONTINUE
206   CONTINUE
C
C A3 = (A**K)*(T**K)/K!
```

```
C
      DO 208 I=1,ORDER
      DO 209 J=1,ORDER
      DO 210 L=1,ORDER
      TEMP(I,J)=TEMP(I,J)+A3(I,L)*A2(L,J)
210   CONTINUE
209   CONTINUE
208   CONTINUE
      DO 211 I=1,ORDER
      DO 212 J=1,ORDER
      A3(I,J)=TEMP(I,J)
      TEMP(I,J)=0
212   CONTINUE
211   CONTINUE
C
C PHI1 = I + AT + (A**2)*(T**2)/2! + . . . (TO KTH ORDER)
C
      DO 213 I=1,ORDER
      DO 214 J=1,ORDER
      PHI1(I,J)=PHI(I,J)+A3(I,J)
214   CONTINUE
213   CONTINUE
C
C IS THE ERROR LESS THAN OR EQUAL TO .1%?
C
      ERROR=0
      DO 215 I=1,ORDER
      ERROR=ERROR+(PHI1(I,I)-PHI(I,I))**2
215   CONTINUE
      IF(ERROR.LE.(.001)) GOTO 218
C
C PHI <-- PHI1
C
      DO 216 I=1,ORDER
      DO 217 J=1,ORDER
      PHI(I,J)=PHI1(I,J)
217   CONTINUE
216   CONTINUE
205   CONTINUE
C
C END OF CONVERGENCE LOOP
C
      IF(OUTPUT.NE.1) GOTO 219
      WRITE(6,100)
      WRITE(6,101)K
      GOTO 219
218   IF(OUTPUT.NE.1) GOTO 219
      WRITE(6,102)K
219   DO 220 I=1,ORDER
      DO 221 J=1,ORDER
      PHI(I,J)=PHI1(I,J)
221   CONTINUE
220   CONTINUE
      IF(OUTPUT.NE.1) GOTO 900
      WRITE(6,103)
      DO 300 I=1,ORDER
      WRITE(6,104) (PHI(I,J),J=1,ORDER)
300   CONTINUE
900   RETURN
      END
      SUBROUTINE CEM(NF,F,NG,G,T,PHI,GAM,T1,T2,T3,IDEN)                 CEM00010
      DIMENSION F(NF,NF),G(NF,NG),PHI(NF,NF),GAM(NF,NG),T1(NF,NF),      CEM00020
     1 T2(NF,NF),T3(NF,NF),IDEN(NF,NF)                                  CEM00030
      REAL IDEN                                                         CEM00040
      DO 10 I=1,4                                                       CEM00050
      DO 20 J=1,NF                                                      CEM00060
20    IDEN(I,J)=0.                                                      CEM00070
10    CONTINUE                                                          CEM00080
      DO 30 I=1,NF                                                      CEM00090
30    IDEN(I,I)=1.                                                      CEM00100
      CALL SMPLY(NF,F,NF,T,T1)                                          CEM00110
      CALL MCOPY(NF,T1,T3)                                              CEM00120
      DO 100 I=1,10                                                     CEM00130
      XN=1./FLOAT(12-I)                                                 CEM00140
      CALL SMPLY(NF,T3,NF,XN,T2)                                        CEM00150
      CALL SMPLY(NF,T2,NF,-1.,T2)                                       CEM00160
      CALL MADD(NF,IDEN,NF,T2,T2)                                       CEM00170
100   CALL MMPLY(NF,T1,NF,T2,NF,T3)                                     CEM00180
      CALL MMPLY(NF,PHI,NF,T2,NF,T3)                                    CEM00190
      CALL MMPLY(NF,T3,NF,G,NG,GAM)                                     CEM00200
      CALL SMPLY(NF,GAM,NG,T,GAM)                                       CEM00210
      RETURN                                                            CEM00220
      END                                                               CEM00230
```

```
C
C
C THIS SUBROUTINE DOES A NONLINEAR SIMULATION USING THE
C RUNGA-KUTTA INTEGRATION TECHNIQUE
C
      SUBROUTINE RKINT(AOA,TC,VEL,ALT,CB,CF,CI,XT,UT)
      DIMENSION FC(8,1),C3(2,4),CF(2,2),CI(2,2),C1(11,1),A1(11,3),
     1 A2(3,18),A(14,8),X(4,1),XNEW(4),XOUT(4),U(2,1),U2(2),UOUT(2),
     2 DY(2,1),DYT(2,1),DX(4),DELX1(4),DELX2(4),DELX3(4),XT(4),UT(2),
     3 DELX4(4),TEMP1(2,1),C2(3,1),FC1(3,1),FC2(18,1),Q0(4),Q1(4),Q2(4),
     4 Q3(4),Q4(4),X1(4),X2(4),X3(4),X4(4),X0(4)
      COMMON/CYCOEF/CY,CY0,CYB,CYDR,CYDA,CYP,CYR
      COMMON/CNCOEF/CN,CN0,CNB,CNDR,CNDA,CNP,CNR
      COMMON/CLCOEF/CL,CL0,CLB,CLDR,CLDA,CLP,CLR
810   FORMAT('-','INPUT: SECONDS FOR TIME HISTORY (INTEGER)')
      ISEC=5
      ITER=10*ISEC
      IOUT=ITER/50
      INDCTR=0
C
C INPUT COMMAND VECTOR
C
803   FORMAT('-','INPUT: COMMAND VECTOR - (Y1,Y2)')
      DY(1,1)=10.
      DY(2,1)=0.
605   DO 115 I=1,2
115   DY(I,1)=DY(I,1)*3.14159/180.
C
C INPUT INITIAL CONDITION
C
      DO 110 I=1,4
110   X(I,1)=XT(I)
C
C COMPUTE INITIAL CONTROL
C
      CALL MMPLY(2,CF,2,DY,1,U)
       DO 112 I=1,2
112   U(I,1)=U(I,1)+UT(I)
C
C SET TIME AND TIME STEP
C
      TIME =0.
      TSTEP=.1
804   FORMAT(' ','TIME',4X,'YAW RATE',3X,'SIDESLIP',3X,'ROLL RATE',
     1         1X,'ROLL ANGLE',3X,'RUDDER',5X,'AILERON')
805   FORMAT(' ','----',4X,'--------',3X,'--------',3X,'---------',
     1         1X,'----------',3X,'------',5X,'-------')
806   FORMAT(' ',F5.2,6(3X,F8.3))
815   FORMAT('-','NONLINEAR TIME HISTORY')
816   FORMAT('0','ALL OUTPUT IN DEGREES OR DEG/SEC')
      WRITE(6,815)
      WRITE(6,816)
      WRITE(6,804)
      WRITE(6,805)
      DO 120 I=1,4
```

```
120   XOUT(I)=(X(I,1)-XT(I))*180./3.14159
      DO 122 I=1,2
122   UOUT(I)=(U(I,1)-UT(I))*180./3.14159
      WRITE(6,806)TIME,(XOUT(I),I=1,4),(UOUT(J),J=1,2)
      DO 130 I=1,4
130   Q0(I)=0.
C
C ITERATE TIME HISTORY
C
      DO 200 K=1,ITER
      INDCTR=INDCTR+1
C
C REDIMENSION X AND U FOR USE IN DNDYN SUBROUTINE
C
      DO 210 I=1,4
210   X0(I)=X(I,1)
      DO 212 I=1,2
212   U2(I)=U(I,1)
C
C DELX1
C
      CALL DNDYN(AOA,TC,VEL,ALT,X0,U2,DX)
      DO 215 I=1,4
215   DELX1(I)=DX(I)*TSTEP
      DO 217 I=1,4
217   X1(I)=X0(I)+.5*DELX1(I)
C
C DELX2
C
      CALL DNDYN(AOA,TC,VEL,ALT,X1,U2,DX)
      DO 220 I=1,4
220   DELX2(I)=DX(I)*TSTEP
      DO 222 I=1,4
222   X2(I)=X0(I)+.5*DELX2(I)
C
C DELX3
C
      CALL DNDYN(AOA,TC,VEL,ALT,X2,U2,DX)
      DO 225 I=1,4
225   DELX3(I)=DX(I)*TSTEP
      DO 227 I=1,4
227   X3(I)=X0(I)+DELX3(I)
C
C DELX4
C
      CALL DNDYN(AOA,TC,VEL,ALT,X3,U2,DX)
      DO 230 I=1,4
230   DELX4(I)=DX(I)*TSTEP
      DO 232 I=1,4
232   X4(I)=X0(I)+(1./6.)*(DELX1(I)+2*DELX2(I)+2*DELX3(I)+DELX4(I))
C
C COMPUTE X(K+1)
C
      DO 235 I=1,4
235   X(I,1)=X4(I)-XT(I)
```

```
C
C COMPUTE NEW TIME
C
      TIME=TSTEP*FLOAT(K)
C
C COMPUTE NEW CONTROL
C
      CALL MMPLY(2,CB,4,X,1,U)
      CALL MMPLY(2,CF,2,DY,1,TEMP1)
      CALL MADD(2,U,1,TEMP1,U)
      DYT(1,1)=DY(1,1)*TIME
      DYT(2,1)=DY(2,1)*TIME
      CALL MMPLY(2,CI,2,DYT,1,TEMP1)
      CALL MADD(2,U,1,TEMP1,U)
      DO 237 I=1,2
237   U(I,1)=U(I,1)+UT(I)
      DO 238 I=1,4
238   X(I,1)=X(I,1)+XT(I)
C
C OUTPUT TIME, X, AND U
C
      IF(INDCTR.NE.IOUT) GOTO 200
      DO 240 I=1,4
240   XOUT(I)=(X(I,1)-XT(I))*180./3.14159
      DO 242 I=1,2
242   UOUT(I)=(U(I,1)-UT(I))*180./3.14159
      WRITE(6,806)TIME,(XOUT(I),I=1,4),(UOUT(J),J=1,2)
      INDCTR=0
200   CONTINUE
811   FORMAT('-','ANOTHER COMMAND VECTOR? (1=YES,0=NO)')
      IF(DY(1,1).EQ.0.) GOTO 610
      DY(1,1)=0.
      DY(2,1)=0.
      GOTO 605
610   RETURN
      END
```

Appendix C

CAS SOFTWARE

The CAS software was developed to implement the control law for actual flight testing aboard the ARA. The software had the following requirements: accept analog inputs on aircraft states and pilot commands; update the gains; compute the control law; and output commands to the control surfaces. The whole software package was limited to 28K of RAM memory in the Micro-DFCS.

The digital flight control system software, pCAS version 6, revision 5, was the CAS program to be implemented on the microprocessor and is presented in Table 15. The software was developed by altering an existing program, pCAS version 4, revision 3 (Ref. 9). It was broken up into four sections--the data stucture declarations, the utility routines, the control routines, and the main program. All software was developed using Pascal-MT.

The data structure declaration section set constant values used in hardware initialization and control law calculations. It also declared variable types (real, integer, array, etc) as required by Pascal-MT.

The utility routines were used for interfacing the software with the hardwwere and are arranged as a set of subroutines.

DELAY10, **DELAY30**, and **Wait1Second** created delays in software execution of 10 microseconds, 30 microseconds, and 1 second, respectively. **COLDBOOT** initialized hardware (clocks, interrupts, input and output ports), zeroed gain matrices and nominal states, and set the gain schedules. **WARMBOOT** zeroed the controls and armed the interrupts. **AnaTEST** enabled the operator to check the A/D and D/A converter operation. **MDISP22** and **MDISP24** displayed 2 by 2 and 2 by 4 matrices, respectively. **MMPLY79** and **MMPLY718** calculated a single gain each using the gain index and the gain coefficient matrices A_1 and A_2, respectively.

The control routines were used for updating the gains and computing the control law. **SETUP** entered the flight condition, computed the flight condition vector, and computed whichever gain was to be updated. It also entered and set the nominal states as required. **CONTROL** entered the current values of the states, computed the perturbations from the nominal condition, computed the control law, and sent the commands to the control surfaces.

The main program performed the background routine for accepting operator inputs and performing a limited number of tasks including: reinitializing, halting, or breaking the program execution; testing the A/D and D/A operations; and resetting the nominal condition.

The program could operate in two modes--flight test or ground test. The flight test mode operated by accepting operator inputs and outputting short messages on CAS operations. The ground test mode performed the flight test functions, and it also executed various steps in the control sequence for error checking.

Table 15. CAS Version 6.5 Listing

```
{****************************************************************
*                                                               *
*         -------------------------------------------           *
*         ! pCAS-1 Digital Flight Control Software System !     *
*         -------------------------------------------           *
*                          VERSION 6.5                          *
*                                                               *
****************************************************************}

Program CAS ;

           {$Z $1700}      {   6K RTP - hardware 9511    }
           {$O $9000}      {     Set Program ORIGIN      }
           {$R $8100}      {    Set location of  RAM     }

           (**********************************)
           (*                                *)
           (*  1.  DATA STRUCTURE DECLARATION *)
           (*                                *)
           (**********************************)


{-------------------------------------------------------------------}
{-------------------- 1.1  Constant Declarations -------------------}
{-------------------------------------------------------------------}
CONST
       ADATA    = $D4;                 { 9511 data port }
       ACONTROL= $D5;                  { 9511 control/status port }

       ADDAM    = $3000;               { memory base for analog board }
       ELEloc   = $3008;               { D/A output location- elevator }
       FLPloc   = $300A;
       ALRloc   = $3010;               { ailerons D/A }
       RUDloc   = $3012;               { rudder D/A }
       SFPloc   = $3014;
       ENGloc   = $3016;

       fm       = 10;                  { for the of TERMIFLEX display }
       prec     = 4;                   { number of significant digits }

       iMASK    = $02;                 { mask for priority interrupt cont }
       iUNMASK  = $03;                 { enable p.i.c. }
       intCONT  = $D7;                 { device code for p.i.c. }
       iJMPloc  = $FFE3;               { RST2 from p.i.c. is vectored to }
                                       {       this location by UFM prom }
       JMPOP    = $C3;                 { 8080/Z80 JMP opcode }

       iZEROv   = $7FF0;               { integer rep of 0 volts to D/A }
       rZEROv   = 2047.0;              { f.p. rep of 0 volts to D/A ($7FF) }

       in2v     = 0.0048852;           { A/D input ==) volts   }
       v2out    = 204.7;               {    volts  ==) D/A out }

       PCreg    = $92;                 { I/O ports command register }
       P8255    = $EB;                 { loc of 8255 control/status port }
       PORTC    = $EA;                 { loc of 8255 data Port C }

       Tcontrol = $DF;                 { timers' control word register }
       T0counter = $DC;                { 8253 timer counter #0 }
       T1counter = $DD;                { 8253 timer counter #1 }

       Udata    = $EC;                 { UART data port number }
       Ucontrl  = $ED;                 { UART control/status port }
       Umode    = $4E;                 { UART mode instruction format }
       Ucommand = $37;                 { UART command instruction format }

       UFM      = $0092;               { re-entry point for UFM monitor }
       Zero     = 0.0;


       THchan  = 1;                    { throttle A/D }
       AOAchan = 26;                   { angle of attack }
       VELchan = 6;                    { velocity }

       RSTchan = 8;                    { roll stick }
       PDchan  = 9;                    { pedals }
```

```
          SSchan  = 11;
          RAchan  = 12;                      { roll angle }
          RRchan  = 13;                      { roll rate }
          YRchan  = 14;                      { yaw rate }

          v2TH    = -0.476;                  { A/D volts ==) TH  cm for'd  }
          v2VEL   =  5.1;                    {     "     ==) VEL ft/sec    }
          v2AOA   =  2.86;                   {     "     ==) AOA deg       }

          v2PD    = -0.256;                  { A/D volts ==) PD  in. forward}
          v2RST   = -8.210;                  {     "     ==) RST deg. clockwise}
          v2YR    = -2.76;                   {     "     ==) YR  deg/sec   }
          v2SS    = -3.08;                   {     "     ==) SS  deg       }
          v2RR    =  4.08;                   {     "     ==) RR  deg/sec   }
          v2RA    = -8.19;                   {     "     ==) RA  deg       }

          RUD2v   =  0.4150;                 { RUD deg right ==)   "       }
          ALR2v   = -0.5100;                 { ALR deg right ==)   "       }

{-----------------------------------------------------------------------}
{--------------------- 1.2  Type Declaration ---------------------------}
{-----------------------------------------------------------------------}
TYPE

          TermLine = PACKED ARRAY [0..11] OF CHAR;

          MAT2x2   = ARRAY [1..2,1..2] OF REAL;
          MAT2x4   = ARRAY [1..2,1..4] OF REAL;
          MAT7x18  = ARRAY [1..7,1..18] OF REAL;
          MAT7x9   = ARRAY [1..7,1..9] OF REAL;
          VEC9     = ARRAY [1..9] OF REAL;
          VEC18    = ARRAY [1..18] OF REAL;

{-----------------------------------------------------------------------}
{--------------------- 1.3  Variable Declarations ----------------------}
{-----------------------------------------------------------------------}
VAR
          chptr  : ^char;
          intptr : ^integer;
          blankline,baden,badprog : termline;
          CR,LF,ACK,END,EOT,comchar : char;
          GroundTest : boolean;
          MASK,CI,CO : integer;

          VEL,AOA,TH,TC,Q,RHO,VEL2,AOA2,TC2,VELck,AOAck    :  real;
          VELnom,AOAnom,THnom,TCck,TCnom                   :  real;
          YR,SS,RR,RA,PD,RST,RSTint,RAC,RAnomC             :  real;
          YRnom,SSnom,RRnom,RAnom,PDnom,RSTnom             :  real;

          RUD,ALR,ELE                                      :  real;
          RUDnom,ALRnom,ELEnom                             :  real;
          ELEout,RUDout,ALRout,index,i,index1,flg          :  integer;
          ELEptr,RUDptr,ALRptr                             :  ^integer;

          Cf,Cs                                            :  MAT2x2;
          Cb                                               :  MAT2x4;
          A1                                               :  MAT7x9;
          A2                                               :  MAT7x18;
          FC1,C1                                           :  VEC9;
          FC2                                              :  VEC18;

{-----------------------------------------------------------------------}
{+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++}
{-----------------------------------------------------------------------}
(*$I Util65.Pas*)


{-----------------------------------------------------------------------}
{+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++}
{-----------------------------------------------------------------------}
(*$I Con65.Pas*)


{-----------------------------------------------------------------------}
{+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++}
{-----------------------------------------------------------------------}

{:::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::}
{!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!    MAIN  PROGRAM    !!!!!!!!!!!!!!!!!!!!!!}
```

```
{=====================================================================}
BEGIN                                                           { CAS }

      chptr    := iJMPloc;                      { Set interrupt jump }
      chptr^   := chr(JMPOP);
      intptr   := iJMPloc+1;
      intptr^  := ord( addr(CONTROL) );

      CI   := addr(CONIN);                      { Define addresses for }
      CO   := addr(CONOUT);                     { -  Redirected I/O    }

      coldboot;                                 { Initialize hardware  }
      warmboot;

      REPEAT                                    (*  MAIN COMMAND LOOP *)

            Read( [CI], Comchar );              {  Wait for a command  }

            CASE Comchar OF                     { Interpret the command}

               'A' :      anaTEST;
               'B' :      INLINE( $F3 / $C3 / UFM );
               'I' :      warmboot;
               'H' :      writeln( [CO], LF,CR, 'CAS offline!' );
               'R' :      BEGIN   flg := 1;
                                  writeln([CO],CR,'RESET NOMS!!')
                          END;
               'V' :      BEGIN   writeln([CO],LF,CR,'UP AND AWAY');
                                  write  ([CO],   CR,'** MODE **');
                                  wait1second
                          END
                   ELSE   BEGIN   writeln( [CO], CR,badem );
                                  wait1second;
                                  warmboot
                          END
            END   { case }

      UNTIL Comchar = 'H';                      { If 'HALT' then exit   }
                                                {  program and halt Z80 }
      INLINE( $F3 / $76 )

END.   { CAS }

{=====================================================================}
{==========================    FINIS  !!!    =========================}
{=====================================================================}
```

```
                    (**************************************)
                    (*                                    *)
                    (*   2.   CAS  Utility  Subroutines   *)
                    (*                                    *)
                    (**************************************)


{-------------------------------------------------------------------}
{----------------------- 2.1  Software Delays ----------------------}
{-------------------------------------------------------------------}
Procedure DELAY10;                       { delay 10 microsecs }
BEGIN
        INLINE($E3/$E3)
END;

Procedure DELAY30;                       { delay 30 microsecs }
BEGIN
        INLINE($E3/$E3/$E3/$E3/$E3/$E3)
END;

Procedure Wait1Second;
VAR     i : integer;
BEGIN
        FOR i := 1 TO 9999 DO delay30
END;   { Wait10sec }
{-----------------------------------------------------------------}
{------------------------ 2.2  Console I/O -----------------------}
{-----------------------------------------------------------------}
Procedure CONOUT( ch : char );
BEGIN
        While Input[UCONTRL] & $0001 = 0  DO delay10;
        Output[UDATA] := ch
END;


Function CONIN  : char;
VAR     chin : char;
        ich  : integer;
BEGIN
        While Input[UCONTRL] & $0002 = 0  DO delay10;
        ich  := ord( Input[UDATA] & $7F );
        chin := chr( ich );
        While Input[UCONTRL] & $0001 = 0  DO delay10;
        Output[UDATA] := chin;
        CONIN := chin
END;

{-------------------------------------------------------------------}
{------------------- 2.3  Program Initialization -------------------}
{-------------------------------------------------------------------}
Procedure COLDBOOT ;

VAR
        i,j,k : integer;
        testmode : char;
        cptr : ^char;
BEGIN
        Output[INTCONT] := chr(iMASK);  { mask interrupt controller }
        INLINE( $ED/$46 );              { set Z80 interrupt mode 0  }
        DISABLE;

        Output[TCONTROL] := chr( $36 ); { initialize TIMER0 }
        Output[T0COUNTER] := chr( $0A );
        Output[T0COUNTER] := chr( $00 );
        delay10;

        Output[TCONTROL] := chr( $74 ); { initialize TIMER1 }
        Output[T1COUNTER] := chr( $10 );
        Output[T1COUNTER] := chr( $27 );
        delay30;

        Output[P8255] := chr( PCREG );  { initialize parallel PORT C }
        Output[PORTC] := chr( $00FF );

        Output[UCONTRL] := chr( $80 );  { initialize UART }
        delay10;
        Output[UCONTRL] := chr( $80 );
        delay10;
        Output[UCONTRL] := chr( $40 );
        delay10;
        Output[UCONTRL] := chr( UMODE );
        delay10;
        Output[UCONTRL] := chr( UCOMMAND );
        delay10;

        cptr := ADDAM;                   { initialize analog board }
        cptr^ := chr( $01 );

        FOR i := 1 TO 2 DO BEGIN         { zero the 2X2 gain matrices }
          FOR j := 1 TO 2 DO BEGIN
                Cf[i,j] := zero;
                Cs[i,j] := zero
          END
        END;

        FOR i := 1 TO 2 DO               { zero the 2X4 gain matrices }
          FOR j := 1 TO 4 DO
                Cb[i,j] := zero;

        FOR i := 1 TO 7 DO               { zero the 7x9 gain schedule }
          FOR j := 1 TO 9 DO
                A1[i,j] := zero;

        FOR i := 1 TO 7 DO               { zero the 7x18 gain schedule }
          FOR j := 1 TO 18 DO
                A2[i,j] := zero;

index:=1;
index1:=1;
flg := 1;

AOAnom:=0.0;
TCnom:=0.0;
VELnom:=0.0;

YRnom:=0.0;
SSnom:=0.0;
RRnom:=0.0;
RAnom:=0.0;
RAnomC:=0.0;
RAC   :=0.0;
PDnom :=0.0;
RSTnom:=0.0;
RSTint:=0.0;

                 { setup gain schedules - A1 and A2  }

 A1[1,1]:=1.79;           A1[1,2]:=-.06704;         A1[1,3]:=.0008409;
 A1[1,4]:=-.007062;       A1[1,5]:=-.0002718;       A1[1,6]:=.000005763;
 A1[1,7]:=.001574;        A1[1,8]:=-.00002029;      A1[1,9]:=.00000005743;

 A1[2,1]:=.7297;          A1[2,2]:=-.02105;         A1[2,3]:=.0002326;
 A1[2,4]:=-.02591;        A1[2,5]:=.0008608;        A1[2,6]:=-.00001064;
 A1[2,7]:=.0007078;       A1[2,8]:=-.00003058;      A1[2,9]:=.000000483;

 A1[3,1]:=2.155;          A1[3,2]:=-.1052;          A1[3,3]:=.00148;
 A1[3,4]:=-.01192;        A1[3,5]:=.0005802;        A1[3,6]:=-.000008376;
 A1[3,7]:=.001896;        A1[3,8]:=-.00007971;      A1[3,9]:=.000001183;

 A1[4,1]:=7.889;          A1[4,2]:=-.3227;          A1[4,3]:=.004372;
 A1[4,4]:=.005739;        A1[4,5]:=.000432;         A1[4,6]:=-.000007653;
 A1[4,7]:=-.001968;       A1[4,8]:=.00003013;       A1[4,9]:=.0000001268;

 A1[5,1]:=3.254;          A1[5,2]:=-.08524;         A1[5,3]:=.000908;
 A1[5,4]:=-.02466;        A1[5,5]:=-.001404;        A1[5,6]:=.0000274;
 A1[5,7]:=.001056;        A1[5,8]:=.0001207;        A1[5,9]:=-.000002462;

 A1[6,1]:=-3.595;         A1[6,2]:=.1556;           A1[6,3]:=-.002146;
 A1[6,4]:=.009345;        A1[6,5]:=-.0006244;       A1[6,6]:=.000009687;
 A1[6,7]:=.0005536;       A1[6,8]:=-.000005157;     A1[6,9]:=-.00000009082;

 A1[7,1]:=.5348;          A1[7,2]:=-.006276;        A1[7,3]:=-.00007038;
 A1[7,4]:=-.1531;         A1[7,5]:=.004123;         A1[7,6]:=-.00004765;
 A1[7,7]:=.003236;        A1[7,8]:=-.000148;        A1[7,9]:=.000002178;

 A2[1,1]:=-1.986;           A2[1,2]:=-.2263;           A2[1,3]:=.06908;
 A2[1,4]:=.11[illegible];   A2[1,5]:=-.0006239;        A2[1,6]:=-.002163;
 A2[1,7]:=-.03[illegible]5; A2[1,8]:=.1234;            A2[1,9]:=.0008762;
 A2[1,10]:=.003228;         A2[1,11]:=-.00001553;      A2[1,12]:=-.00007741;
```

```
A2[1,16]:=-.0001424;     A2[1,17]:=.000001853;     A2[1,18]:=.000003768;

A2[2,1]:=-.05978;        A2[2,2]:=-.1526;          A2[2,3]:=.001118;
A2[2,4]:=.009144;        A2[2,5]:=-.00001153;      A2[2,6]:=-.0001301;
A2[2,7]:=.008944;        A2[2,8]:=-.000245;        A2[2,9]:=-.000395;
A2[2,10]:=-.00007989;    A2[2,11]:=.000006372;     A2[2,12]:=.000001957;
A2[2,13]:=.0004981;      A2[2,14]:=-.0001815;      A2[2,15]:=-.00003247;
A2[2,16]:=.000005954;    A2[2,17]:=.0000003536;    A2[2,18]:=.0000000498;

A2[3,1]:=-.1887;         A2[3,2]:=-.6572;          A2[3,3]:=.005632;
A2[3,4]:=.03318;         A2[3,5]:=-.00009288;      A2[3,6]:=-.0004252;
A2[3,7]:=.09175;         A2[3,8]:=-.03717;         A2[3,9]:=-.001522;
A2[3,10]:=-.001522;      A2[3,11]:=.00002116;      A2[3,12]:=.00002149;
A2[3,13]:=-.00384;       A2[3,14]:=.00266;         A2[3,15]:=-.00003302;
A2[3,16]:=.00005423;     A2[3,17]:=.0000003215;    A2[3,18]:=.0000002887;

A2[4,1]:=-.9162;         A2[4,2]:=-3.385;          A2[4,3]:=-.01312;
A2[4,4]:=.155;           A2[4,5]:=.0003222;        A2[4,6]:=-.002074;
A2[4,7]:=.1234;          A2[4,8]:=.3041;           A2[4,9]:=-.003296;
A2[4,10]:=-.006569;      A2[4,11]:=.00004196;      A2[4,12]:=.00004993;
A2[4,13]:=-.006807;      A2[4,14]:=.001183;        A2[4,15]:=.0001429;
A2[4,16]:=.00003287;     A2[4,17]:=-.0000023;      A2[4,18]:=.0000009122;

A2[5,1]:=-.4223;         A2[5,2]:=.943;            A2[5,3]:=.02269;
A2[5,4]:=-.04981;        A2[5,5]:=-.000333;        A2[5,6]:=.0007071;
A2[5,7]:=-.06199;        A2[5,8]:=.1051;           A2[5,9]:=.001981;
A2[5,10]:=-.003469;      A2[5,11]:=-.00002631;     A2[5,12]:=.00004469;
A2[5,13]:=.002308;       A2[5,14]:=-.00517;        A2[5,15]:=-.00004179;
A2[5,16]:=.0001639;      A2[5,17]:=.0000005309;    A2[5,18]:=-.00000221;

A2[6,1]:=-.658;          A2[6,2]:=1.412;           A2[6,3]:=.03657;
A2[6,4]:=-.06358;        A2[6,5]:=-.000501;        A2[6,6]:=.0008178;
A2[6,7]:=-.08771;        A2[6,8]:=.0682;           A2[6,9]:=.001415;
A2[6,10]:=.00008433;     A2[6,11]:=-.00002001;     A2[6,12]:=-.000002753;
A2[6,13]:=.002882;       A2[6,14]:=-.003604;       A2[6,15]:=.00007385;
A2[6,16]:=-.00001119;    A2[6,17]:=-.0000008737;   A2[6,18]:=-.0000005247;

A2[7,1]:=-8.224;         A2[7,2]:=.6892;           A2[7,3]:=.3341;
A2[7,4]:=.001572;        A2[7,5]:=-.004504;        A2[7,6]:=-.0001901;
A2[7,7]:=.0363;          A2[7,8]:=-.05388;         A2[7,9]:=-.002292;
A2[7,10]:=.001104;       A2[7,11]:=.00003346;      A2[7,12]:=-.000008684;
A2[7,13]:=.001139;       A2[7,14]:=.002541;        A2[7,15]:=.00000266;
A2[7,16]:=-.00007;       A2[7,17]:=-.0000007202;   A2[7,18]:=.00001551;

        RHO     := 0.001946;

        ELEptr := ELEloc;                 { set pointers to D/A locations }
        ALRptr := ALRloc;
        RUDptr := RUDloc;

        CR := chr($0D);  ACK := chr($0A);  ENQ := chr($05);
        LF := chr($0A);  EOT := chr($04);

        writeln( [CO], CR,'Welcome to..');
        write  ( [CO], CR,')) pCAS  ((');
        wait1second;

        writeln( [CO], CR,'Ground test ');
        write  ( [CO], CR,' or Flight?');
        read   ( [CI], testmode );

        IF testmode='G' THEN GroundTest := TRUE
                        ELSE GroundTest := FALSE;

END;  { COLDBOOT }
{-------------------------------------------------------------------}
{-------------------- 2.4  Program Reinitialization -----------------}
{-------------------------------------------------------------------}
Procedure WARMBOOT ;

VAR     iptr : ^integer;
        cptr : ^char;

{ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - }
        Procedure ZOUT( ip : integer );
        VAR    iptr2 : ^integer;
        BEGIN
                iptr2 := ip;
        END;
{ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - }

BEGIN
        DISABLE;
        blankline :=  '                 ';
        baden     :=  ' Entry Error';
        badprog   :=  'Progra Error';

        writeln( [CO], CR,blankline);
        writeln( [CO], CR,'ABHIRV   ?');
        write  ( [CO], CR,'    Option ?');

        MASK := $FFFF;

        ZOUT( ELEloc );                  { zero volts out to controls }
        ZOUT( ALRloc );
        ZOUT( RUDloc );

        Output[INTCONT] := chr(iUNMASK);    { re-enable p.i.c. }
        ENABLE

END;   { WARMBOOT }
{-----------------------------------------------------------------------------}
{-------------------- 2.5  Analog to Digital Conversion ---------------------}
{-----------------------------------------------------------------------------}
Function ADC(Vchan : integer) : real;

VAR     status : char;
        cptr : ^char;
        iptr : ^integer;
        Istatus,Ivar : integer;
BEGIN
        cptr  := ADDAM+1;
        cptr^ := chr(Vchan);
        cptr  := ADDAM;
        cptr^ := chr($01);
        iptr  := ADDAM+4;
        REPEAT
           status  := cptr^;
           istatus := ord(status)
        UNTIL TSTBIT(istatus,7);
        Ivar := iptr^;
        Ivar := SHR( Ivar,4 );
        ADC := Ivar                      { implicit conversion to REAL }
END;   { ADC }
{-----------------------------------------------------------------------------}
{---------------------- 2.6  A/D and D/A Verification -----------------------}
{-----------------------------------------------------------------------------}

Procedure AnaTEST;

VAR     ach : char;

{ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -}
        Procedure testA2D;
        VAR     INchan : integer;
                voltsIN : real;
        BEGIN
                writeln( [CO],CR,' 0..9..F  ?');
                write([CO],CR,blankline);
                write([CO],CR,'Chan In = ?');
                read([CI],ach);

        IF (ach)='0') and (ach(='9') THEN inchan:=ord(ach)-ord('0')
         ELSE IF (ach)='A') and (ach(='F') THEN inchan:=ord(ach)-ord('A')+10
          ELSE BEGIN writeln([CO],CR,baden);
                  wait1second;
                  warmboot;
                  exit
              END;

            REPEAT
               VoltsIn:=(ADC(In[illegible])-r2[illegible])*[illegible];
               writeln([CO],[illegible],' Volts= ');
               write([CO],[illegible]);
               read([CI],a[illegible]
            UNTIL (ach <> [illegible])
```

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HIGH-ANGLE-OF-ATTACK FLIGHT(U) AIR FORCE INST OF TECH
WRIGHT-PATTERSON AFB OH W A EHRENSTROM MAR 83

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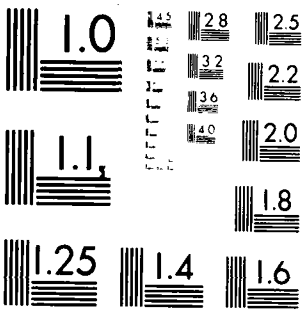

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963-A

```
{-----------------------------------------}
        Procedure testD2A;
        VAR     iptr : ^integer;
                iVoltsOUT : integer;
                VoltsOUT : real;
        BEGIN
                writeln([CO],CR,'    0..5');
                write  ([CO],CR,blankline);
                write  ([CO],CR,'CHAN OUT ? ');
                read   ([CI],ach);
                writeln([CO],CR,'-10..0..+10?');
                write  ([CO],CR,blankline);
                write  ([CO],CR,'Volts=');
                readln ([CI],VoltsOUT);
                iVoltsOUT := ROUND( (VoltsOUT*v2out) + rZEROv );

                CASE ach OF
                   '0' :   iptr := ELEloc;
                   '2' :   iptr := ALRloc;
                   '3' :   iptr := RUDloc;
                END;

                iptr^ := SHL(iVoltsOUT,4) & MASK

        END;
{-----------------------------------------}

BEGIN
        DISABLE;
        writeln([CO],CR,'A/d or D/a');
        write([CO],CR,blankline);
        write([CO],CR,'Which test?');
        read ([CI],ach);

        CASE ach OF
          'A'  :   testA2D;
          'D'  :   testD2A
                ELSE BEGIN      writeln([CO],CR,baden);
                                waitisecond;
                                warmboot;
                                exit
                   END
        END;

        read([CI],ach);
        warmboot
END;

{----------------------------------------------------------------}
{---------------------- 2.7 Matrix Manipulation ------------------}
{----------------------------------------------------------------}

Procedure MDISP22( VAR A : MAT2x2 );

VAR     i,j : integer;
BEGIN
        write([CO],CR);
        FOR i := 1 TO 2 DO BEGIN
          FOR j := 1 TO 2 DO write([CO],A[i,j]:fw:prec,' ');
          writeln([CO],CR)
        END
END;

Procedure MDISP24( VAR A : MAT2x4 );

VAR     i,j : integer;
BEGIN
        write([CO],CR);
        FOR i := 1 TO 2 DO BEGIN
          FOR j := 1 TO 4 DO write([CO],A[i,j]:fw:prec,' ');
          writeln([CO],CR)
        END
END;

Procedure MMPLY718(VAR A:MAT7x18; VAR [illegible] C:[illegible]; VAR i:integer);

BEGIN
        C[i]:=A[i,1]*B[1]   +A[i,2]*[illegible]   +
              A[i,4]*B[4]   +A[i,5]*[illegible]   +
                A[i,10]*B[10] +A[i,11]*B[11] +A[i,12]*B[12] +
                A[i,13]*B[13] +A[i,14]*B[14] +A[i,15]*B[15] +
                A[i,16]*B[16] +A[i,17]*B[17] +A[i,18]*B[18] ;
END;

Procedure MPLY79 (VAR A:MAT7x9; VAR B:VEC9; VAR C:VEC9; VAR i:integer);

BEGIN
        C[i]:=A[i,1]*B[1]  +A[i,2]*B[2]  +A[i,3]*B[3]  +
              A[i,4]*B[4]  +A[i,5]*B[5]  +A[i,6]*B[6]  +
              A[i,7]*B[7]  +A[i,8]*B[8]  +A[i,9]*B[9]  ;
END;


A>
```

```
(***************************************)
(*                                     *)
(*    3.  CAS  Control  Subroutines    *)
(*                                     *)
(***************************************)


{-----------------------------------------------------------------------}
{-------------------- 3.1  Background Setup Routine ---------------------}
{-----------------------------------------------------------------------}
Procedure SETUP ;

{ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - }
        Procedure SET_LAT;

        VAR     i : integer;
                ETC,ETC1,ETC2 : real;

        BEGIN

          TCnom  := (ADC(THchan) - rZEROv) * in2v ;
          AOAnom := (ADC(AOAchan) - rZEROv) * in2v ;
          VELnom := (ADC(VELchan) - rZEROv) * in2v ;

          IF GroundTest THEN BEGIN
          IF index = 100 THEN BEGIN

          write([CO],LF,CR,'AOA');
          write([CO],AOAnom:fw:prec,' ');
          write([CO],LF,CR,'VEL');
          write([CO],VELnom:fw:prec,' ');
          write([CO],LF,CR,'TC');
          write([CO],TCnom:fw:prec,' ');

          END;
          END;

          VELnom := (VELnom * v2VEL + 100.0)*(6.0/3.6);
          AOAnom := (AOAnom * v2AOA + 13.7);
          TCnom  := (TCnom  * v2TH + 4.53)*(0.23/9.2);
          Q    := .5 * VELnom * VELnom * RHO;

                      { set up flight condition vector }

          ETC1:= .001 * TCnom;
          ETC2:= ETC1 * ETC1;
          ETC := 1.0 + ETC1 + ETC2/2.0 + (ETC2*ETC1)/6.0 +
                 (ETC2*ETC2)/24.0 + (ETC2*ETC2*ETC1)/120.0 +
                 (ETC2*ETC2*ETC2)/720.0 ;

     FC1[1]:=ETC;              FC1[2]:=FC1[1]*Q;         FC1[3]:=FC1[2]*Q;
     FC1[4]:=FC1[1]*AOAnom;    FC1[5]:=FC1[2]*AOAnom;    FC1[6]:=FC1[3]*AOAnom;
     FC1[7]:=FC1[4]*AOAnom;    FC1[8]:=FC1[5]*AOAnom;    FC1[9]:=FC1[6]*AOAnom;

     FC2[1]:=1.0;              FC2[2]:=FC2[1]*TCnom;     FC2[3]:=FC2[1]*Q;
     FC2[4]:=FC2[2]*Q;         FC2[5]:=FC2[3]*Q;         FC2[6]:=FC2[4]*Q;
     FC2[7]:=FC2[1]*AOAnom;    FC2[8]:=FC2[2]*AOAnom;    FC2[9]:=FC2[3]*AOAnom;
     FC2[10]:=FC2[4]*AOAnom;   FC2[11]:=FC2[5]*AOAnom;   FC2[12]:=FC2[6]*AOAnom;
     FC2[13]:=FC2[7]*AOAnom;   FC2[14]:=FC2[8]*AOAnom;   FC2[15]:=FC2[9]*AOAnom;
     FC2[16]:=FC2[10]*AOAnom;FC2[17]:=FC2[11]*AOAnom; FC2[18]:=FC2[12]*AOAnom;

                      { calculate gain matrices }

        IF index1 = 1 THEN BEGIN

          i := 1;
          MMPLY79(A1,FC1,C1,i);
          Cb[1,1]:=C1[i];

        END;

        IF index1 = 2 THEN BEGIN

          i := 1;
          MMPLY718(A2,FC2,C1,i);
          Cb[1,2]:=C1[i];

        END;
```

```
IF index1 = 3 THEN BEGIN

  i := 2;
  MMPLY718(A2,FC2,C1,i);
  Cb[1,3]:=C1[i];

END;

IF index1 = 4 THEN BEGIN

  i := 3;
  MMPLY718(A2,FC2,C1,i);
  Cb[1,4]:=C1[i];

END;

IF index1 = 5 THEN BEGIN

  i := 5;
  MMPLY718(A2,FC2,C1,i);
  Cf[1,1]:=C1[i];

END;

IF index1 = 6 THEN BEGIN

  i := 5;
  MMPLY79(A1,FC1,C1,i);
  Cf[1,2]:=C1[i];

END;

IF index1 = 7 THEN BEGIN

  i := 6;
  MMPLY718(A2,FC2,C1,i);
  Ca[1,1]:=C1[i];

END;

IF index1 = 8 THEN BEGIN

  i := 2;
  MMPLY79(A1,FC1,C1,i);
  Cb[2,1]:=C1[i];

END;

IF index1 = 9 THEN BEGIN

  i := 4;
  MMPLY718(A2,FC2,C1,i);
  Cb[2,2]:=C1[i];

END;

IF index1 = 10 THEN BEGIN

  i := 3;
  MMPLY79(A1,FC1,C1,i);
  Cb[2,3]:=C1[i];

END;

IF index1 = 11 THEN BEGIN

  i := 4;
  MMPLY79(A1,FC1,C1,i);
  Cb[2,4]:=C1[i];

END;

IF index1 = 12 THEN BEGIN

  i := 6;
  MMPLY79(A1,FC1,C1,i);
  Cf[2,1]:=C1[i];

END;
```

```
          IF index1 = 13 THEN BEGIN

            i := 7;
            MMPLY79(A1,PC1,C1,i);
            Cf[2,2]:=C1[i];

          END;

          IF index1 = 14 THEN BEGIN

            i := 7;
            MMPLY718(A2,PC2,C1,i);
            Cs[2,1]:=C1[i];

          index1 := 0;

          END;

          index1 := index1 + 1;

            IF GroundTest THEN BEGIN
            IF index = 100 THEN BEGIN

                    write([CO],LF,CR,'  AOA:');
                    write([CO],AOAnom:fw:prec,'  ');
                    write([CO],LF,CR,'  TC:');
                    write([CO],TCnom:fw:prec,'  ');
                    write([CO],LF,CR,'  Q:');
                    write([CO],Q:fw:prec,'  ');
                    write([CO],LF,CR,'  VEL:');
                    write([CO],VELnom:fw:prec,'  ');

                    writeln([CO],CR);
                    writeln([CO],CR,'  Cf:');
                    mdisp22( Cf );
                    writeln([CO],CR,'  Ci:');
                    mdisp22( Cs );
                    writeln([CO],CR,'  Cb:');
                    mdisp24( Cb );

            END;
            END;

          END;  { set_lat }
 { - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - }

 BEGIN                                                            { SETUP }

          IF flg = 1 THEN BEGIN

          YRnom  := (ADC(YRchan)-rZEROv)*in2v;
          YRnom  := (v2YR * YRnom);
          SSnom  := (ADC(SSchan) - rZEROv)*in2v;
          SSnom  := (v2SS * SSnom);
          RRnom  := (ADC(RRchan) - rZEROv)*in2v;
          RRnom  := (v2RR * RRnom);
          RAnom  := (ADC(RAchan) - rZEROv)*in2v;
          RAnom  := (v2RA * RAnom);

          PDnom  := (ADC(PDchan) - rZEROv)*in2v;
          PDnom  := (v2PD * PDnom);
          PDnom  := PDnom * (15.0/2.38);              (scale factor)
          RSTnom := (ADC(RSTchan) - rZEROv)*in2v;
          RSTnom := (v2RST * RSTnom);
          RSTnom := RSTnom * (50.0/80.0);             (scale factor)

          RAC := 0.0;

          flg := 0;

          END;

          IF GroundTest THEN BEGIN
          IF index = 100 THEN BEGIN

          write([CO],LF,CR,'   YRnom');
          write([CO],YRnom:fw:prec);
          write([CO],LF,CR,'   SSnom');
          write([CO],SSnom:fw:prec);
          write([CO],LF,CR,'   RRnom');
```

```
        write([CO],RRnom:fw:prec);
        write([CO],LF,CR,'    RAnom');
        write([CO],RAnom:fw:prec);
        write([CO],LF,CR,'    PDnom');
        write([CO],PDnom:fw:prec);;
        write([CO],LF,CR,'    RSTnom');
        write([CO],RSTnom:fw:prec);

        END;
        END;

        Set_Lat;

END;  { SETUP }

{-----------------------------------------------------------------------}
{-------------------- 4.2  Flight Control Routines ---------------------}
{-----------------------------------------------------------------------}

Procedure  CONTROL;                                  { Interrupt Service }

CONST   MINvolts = $0000;
        MAXvolts = $0FFF;

BEGIN

        INLINE( $F5/$3E/$02/$D3/$D7/$E3/$E3/$E3/$E3/$E3/$E3/$F1 );
        DISABLE;
        INLINE( $08/$D9 );                      { Z-80 Register Exchanges }

        ELEout := MAXvolts;
        ELEptr^ := SHL(ELEout,4) & MASK;

{ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - }

                SETUP;

                YR  := (ADC(YRchan) - rZEROv)*in2v;
                YR  := (v2YR * YR);
                SS  := (ADC(SSchan) - rZEROv)*in2v;
                SS  := (v2SS * SS);
                RR  := (ADC(RRchan) - rZEROv)*in2v;
                RR  := (v2RR * RR);
                RA  := (ADC(RAchan) - rZEROv)*in2v;
                RA  := (v2RA * RA);

                PD  := (ADC(PDchan) - rZEROv)*in2v;
                PD  := (v2PD * PD);
                PD  := PD * (15.0/2.38);
                RST := (ADC(RSTchan) - rZEROv)*in2v;
                RST := (v2RST * RST);
                RST := RST * (50.0/80.0);

        IF GroundTest THEN BEGIN
        IF index = 100 THEN BEGIN

        write([CO],LF,CR,'    YR:');
        write([CO],YR:fw:prec);
        write([CO],LF,CR,'    SS:');
        write([CO],SS:fw:prec);
        write([CO],LF,CR,'    RR:');
        write([CO],RR:fw:prec);
        write([CO],LF,CR,'    RA:');
        write([CO],RA:fw:prec);
        write([CO],LF,CR,'    PD:');
        write([CO],PD:fw:prec);
        write([CO],LF,CR,'    RST:');
        write([CO],RST:fw:prec);

        END;
        END;

        YR := YRnom - YR;
        SS := SSnom - SS;
        RR := RRnom - RR;
        RA := RAnom - RA;

        PD := PDnom - PD;
        RST:= RSTnom - RST;
```

```
RSTint := RAC + RST * .1;
RAC    := RSTint;

RUD := Cf[1,1]*RST+Cf[1,2]*PD+Cs[1,1]*RSTint;
RUD := RUD + (Cb[1,1]*YR+Cb[1,2]*SS+Cb[1,3]*RR+Cb[1,4]*RA);

ALR := Cf[2,1]*RST+Cf[2,2]*PD+Cs[2,1]*RSTint;
ALR := ALR + (Cb[2,1]*YR+Cb[2,2]*SS+Cb[2,3]*RR+Cb[2,4]*RA);
ALR := ALR/2.0;

IF GroundTest THEN BEGIN
IF index = 100 THEN BEGIN

write([CO],LF,CR,'  RUD');
write([CO],RUD:fw:prec);
write([CO],LF,CR,'  ALR');
write([CO],ALR:fw:prec);

END;
END;

RUD := RUD2v * RUD;
RUD := (RUD * v2out) + rZEROv;
ALR := ALR2v * ALR;
ALR := (ALR * v2out) + rZEROv;

IF GroundTest THEN BEGIN
IF index = 100 THEN BEGIN

write([CO],LF,CR,'  RUDvolts');
write([CO],RUD:fw:prec);
write([CO],LF,CR,'  ALRvolts');
write([CO],ALR:fw:prec);

index := 0;

END;
END;

index := index + 1;

                IF (RUD<=zero)  THEN RUDout := MINvolts
                                ELSE RUDout := Trunc(RUD);
                IF (RUDout > MAXvolts) THEN RUDout := MAXvolts;

                IF (ALR<=zero)  THEN ALRout := MINvolts
                                ELSE ALRout := Trunc(ALR);
                IF (ALRout > MAXvolts) THEN ALRout := MAXvolts;

                RUDptr^ := SHL(RUDout,4) & MASK;
                ALRptr^ := SHL(ALRout,4) & MASK;


{ - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -}

        ELEout := MINvolts;
        ELEptr^ := SHL(ELEout,4) & MASK;

        INLINE( $D9/$D8 );
        INLINE( $F5/$3E/$03/$D3/$D7/$F1 );
        ENABLE

END;  { CONTROL }

A)
```

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